IMAGE SINGULARITY DETECTION USING MULTISCALE TRANSFORMS

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Abstract

In images most of the information is often carried by transients and edges. The local maxima of multiscale transform modulus detect the location of irregular structures. In this paper a new approach is used for singularity detection which is based multiscale transform. Multiscale wavelet transforms and the directional Fast Discrete Curvelet transform (FDCT) is used to detect the fine edges. FDCT is constructed by wrapping, parabolic scaling of Fourier coefficients named in two dimensions. It gives better sparsity. As an example for image enhancement image denoising shows better results by preserving edges for directional multiscale transform than multiscale wavelet transform.

Keywords - Singularity, Denoising, Multiscale Transform Curvelet Transform.

1 INTRODUCTION

Singularity is defined as any discontinuity in a smooth area. Singularities and irregular structures often carry the most important information in signals. It finds minority part in an image, which may be edges or discontinuities across edges. In images, the discontinuities of the intensity provide the location of the object counter, which are particularly meaningful for recognition purpose. Fourier transform was the main mathematical tool for analyzing singularities. The Fourier transform is global and provides a description of the overall regularity of the signal. But it is not well adapted for finding the location and the spatial distribution of singularities. Short Time Fourier transform gives frequency component of particular time interval. Hence it somewhat obeys Heisenberg's principle and effectively track these singularity as it u–ses a fix window. Multiscale ideas with Wavelet transform are well adapted for finding the location and spatial distribution of singularities. In Wavelet the discontinuities across curves are spatially distributed because of this they can interact rather extensively with many terms in the wavelet expansion [3]. So the Wavelet representation is not sparse. All singularities in one dimension are point singularities so Wavelets have a certain universality [5,6]. In higher dimensions there are more types of singularities and Wavelet loses their universality. Wavelets do well for point singularities and not for singularities along curves. Wavelets are not well adapted to edges because of its isometric scaling. To represent the singularities along curves, 2D multiscale transforms such as Contourlet transform, Curvelet transform are more useful [9]. These directional transforms represents the discontinuity along the edges. Fast Discrete Curvelet Transform (2D FDCT) track curves with tight frame. The concept of anisotropic principle and parabolic scaling has been applied for efficient detection of singularities by providing coefficients with directional features FDCT. In this paper a image denoising example is considered to show the singularities by using wavelet and Fast Discrete Curvelet transform.

The paper is organized as follows. Section II describes the singularity detection. Curvelet transform is explained in section III. Section IV gives the results for the application of image denoising. Section V gives the conclusion.

2 SINGULARITY DETECTION

To detect the singularities, an object $f(x_1, x_2)$ which is smooth apart from a singularity along a planar curve η, η could trace out a circle in the plane and f could be discontinuous along η, a specific case being $f(x) = 1_{\{|x| \le 1\}}$ which is discontinuous at the unit circle. In imaging applications, where η represents an edge in the image f. In the analysis of singularities, wavelet transform resolve the singular support of f.Using an appropriate wavelet ψ the classic continuous wavelet transform $CW_f(a,b) = \langle \psi_{a,b}, f \rangle$ will signal the location of the singularity through its asymptotic behavior as the scale $a \to 0$. For each fixed location $x_0, CW_f(a, x_0)$ typically will tend to zero rapidly for x_0 outside the singularity and typically will tend to zero slowly on the singularity. Thus the location of slow decay for the wavelet transform are the points where f is singular [5]. For the directional transform, with parameters (a, b, θ) and the asymptotic behavior as $a \to 0$ for (b, θ) fixed, have a rapid decay for b away from the singularity, but will have slow decay in all directions θ at points b on the singularity. Thus the asymptotic behavior of the transform as $a \to 0$ for the b fixed is unable to indicate clearly the true underlying directional phenomenon which is a singularity having a precise orientation at a specific location. The transform defined here has the property that if the singularity is a curve, then for fixed $(x_0, \theta_0), \Gamma_f(a, x_0, \theta_0)$ will tend to zero rapidly as $a \to 0$ unless (x_0, θ_0) matches both the location and orientation of the singularity. In this paper Curvelet transform is used to track the singularities along the curve. The geometry of the transform with anisotropic scaling and tight framing gives better representation of the singularity along curve.

3 FAST DISCRETE CURVELE TRANSFORM

Fast Discrete Curvelet transform (FDCT) gives local components at different frequencies for analysis and synthesis of digital image in multi-resolution analysis. FDCT is multi-scale geometric transform, which is a multi-scale pyramid with many directions and positions at each length scale [1]. FDCT is basically 2D anisotropic extension to classical wavelet transform that has main direction associated with it. Analogous to wavelet, FDCT can be translated and dilated. The dilation is given by a scale index that controls the frequency content of the Curvelet with the indexed position and direction can be changed through a rotation. This rotation is indexed by an angular index. Curvelet satisfy anisotropic scaling relation, which is generally referred as parabolic scaling. This anisotropic scaling relation associated with FDCT is a key ingredient to the proof that Curvelet provides sparse representation of the C^2 function away from edges along piecewise smooth curves [2, 3]. FDCT is constructed by a radial window W and angular window V. The radial window W is expressed as

$$\tilde{W}_j(w) = \sqrt{\phi_{j+1}^2(w) - \phi_j^2(w)} \quad , \quad j \ge 0 \tag{1}$$

Where, j is scale and ϕ is defined as the product of low-pass one dimensional window and separate scales in Cartesian equivalents. The angular window V is defined as

$$V_j(w) = V\left(2^{\lfloor j/2 \rfloor} w_2 / w_1\right) \tag{2}$$

where, $W1$ and $W2$ are low pass one dimensional windows.

The Cartesian window $\tilde{U}_{j,l}$ is constructed by combining radial window W and angular window V and is expressed as

$$\tilde{U}_{j,l}(w) = W_j(w) V_j(S_{\theta l} w) \tag{3}$$

Where, the angle θ_l have same slope but are not equally spaced. S_θ is shear matrix, $S_\theta = \begin{pmatrix} 1 & 0 \\ \tan\theta 1 \end{pmatrix}$. Shear matrix S_θ is used to maintain the symmetry around the origin and rotation by $\pm\prod/2$ radiance. The family $\tilde{U}_{j,l}$ implies a concentric tiling whose geometry is shown in Fig. 1. The shaded region represents a wedge.

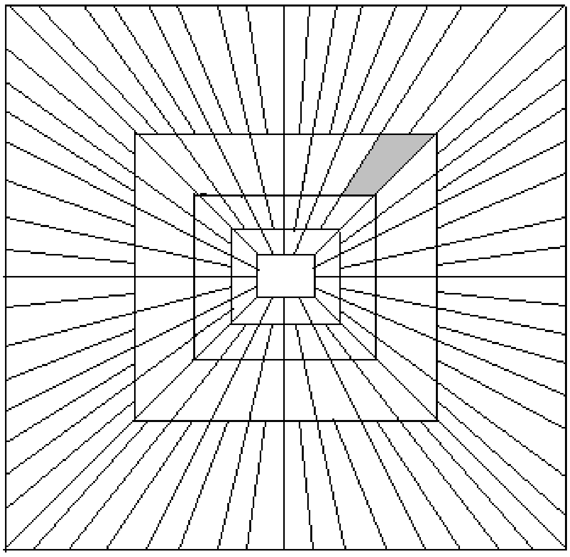

Fig.1. Basic Digital tiling.

The above construction gives pseudo-polar tiling, an alternative to ideal polar tiling. In FDCT via wrapping the Curvelets at a wedge are wrapped for a given scale and angle by translating the Curvelet on regular rectangular grid which is same for every angle within each quadrant with proper orientation. The frequency domain definition of digital Curvelet is,

$$\overline{\varphi^D_{j,l,k}}[t_1, t_2] = \hat{U}_j[t_1, t_2] e^{-i2\Pi[k_1 t_1 + k_2 t_2]} \quad (4)$$

where, $\hat{U}_j[t_1, t_2]$ is Cartesian window. Here the discrete localizing window $\hat{U}_{j,l}[n_1, n_2]$ does not fit in a rectangle, aligned with axes. At each scale j, there exist two constants $L_{1,j} \approx 2^j$ and $L_{2,j} \approx 2^{j/2}$ such that for every orientation θ_l, one can tile the two-dimensional plane which translates the respective rectangle by multiples of $L_{1,j}$ in the horizontal direction and $L_{2,j}$ in the vertical direction. The windowed data is wrapped around the origin. The correspondence between the wrapped and original indices is one to one where the wrapping transformation is re-indexing of the data. Fig. 2 illustrates the wrapping process. Discrete Curvelet transform is expressed as

$$c^D(j, l, k) = \sum_{0 \le t_1, t_2 < n} f[t_1, t_2] \overline{\varphi^D_{j,l,k}}[t_1, t_2] \quad (5)$$

where, $c^D(j, l, k)$ represents Curvelet coefficients with *j* as scale parameter, *l* as orientation parameter and *k* as position parameter. $f[t_1, t_2]$ is an input of Cartesian arrays [1]. This transform is also invertible.

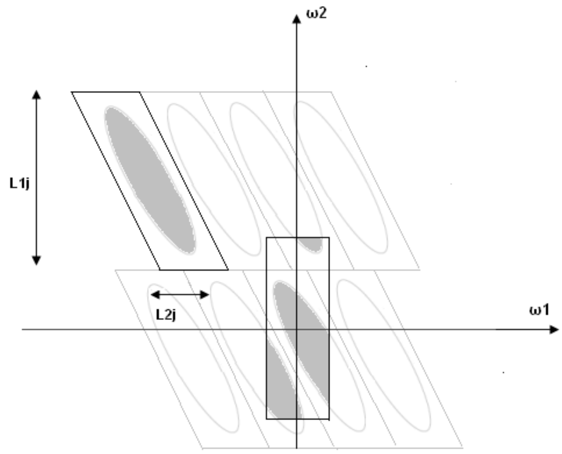

Fig.2. Illustrate the wrapping process

4 RESULTS AND DISCUSSION

As an application for image enhancement, image denoising is performed on a "Lena" test image of size 256×256 pixels with Gaussian noise $\sigma = 10$. For comparison purpose the result of wavelet transform and FDCT on same test image is considered. The result of the test image is shown in Figure 3. Fig. 3(a) shows the image denoised with wavelet transform and Fig. 3 (b) shows image denoised by using FDCT. The quality of denoising is evaluated using the mean squared error (MSE) and peak signal to noise ratio (PSNR) in dB. The results with the Fast Discrete Curvelet transform have improved a lot. The FDCT denoised the image significantly while preserving all the edge details. The details of the eye balls, hairs and the curvature of the hat shows sharp transitions as compared to the image denoised by using wavelet transform. The denoised edges of the image are sharp. The MSE and PSNR of the test image with FDCT are 77.9992 and 29.2099dB where as the wavelet transform is 273.2419 and 23.7653dB. The MSE of the test image with Curvelet transform is reduced much more. The MSE and PSNR is evaluated using following formula:

$$MSE = \frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} \left(x(i,j) - p(i,j) \right)^2 \tag{6}$$

$$PSNR = 10 \log \left[\frac{(255)^2}{MSE} \right] db \tag{7}$$

where $x(i,j)$ is original image and $p(i,j)$ is denoised image.

Fig.3. (a) Denoised image with Wavelet transform

Fig.3 (b) Denoised image with Curvelet transform

5 CONCLUSION

The Curvelet transform tracks well the singularities along the smooth curve with sparse representation. It denoises the image well preserving all the edge details and singularities. The result shows appropriate improvement in MSE and PSNR with a directional transform. The singularities of an image have been well detected by FDCT, a directional multiscale transform as compared to conventional wavelet transform.

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