ACYCLIC EDGE-COLORING OF HEXAGONAL, HONEYCOMB AND SIERPINSKI NETWORKS

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***Abstract*: In this paper, we find the minimum acyclic edge chromatic number of Hexagonal network, Honeycomb network and Sierpinski networks.

***Keywords*:** acyclic edge-coloring, acyclic chromatic index, hexagonal network, honeycomb network, sierpinski network.

1.INTRODUCTION

All graphs we consider are simple and finite. Let $\Delta(G)$ denote the maximum degree of a graph G. A coloring of the edges of a graph is *proper* if no pair of incident edges receive the same color. A proper coloring C of the edges of a graph G is *acyclic* if there is no two-colored (bichromatic) cycle in G with respect to C. The minimum number of colors required to edge-color a graph G acyclically is termed the *acyclic chromatic index* of G and is denoted by $a'(G)$. The notion of acyclic coloring was introduced by Grunbaum in [6]. Determining $a'(G)$ either theoretically or algorithmically has been a very difficult problem. Even for the highly structured and simple class of complete graphs, the value of $a'(G)$ is not yet determined. Determining the exact values of $a'(G)$ even for very special classes of graphs is still open [3]. The acyclic chromatic index and its vertex analogue can be used to bound other parameters like oriented chromatic number and star chromatic number of a graph G, both of which has many practical applications such as in wavelength routing in optic networks [8]. In other words, in all-optical networks a single physical optical link can carry several logical signals provided that they are transmitted on different wavelengths. An all to all routing in n- node network is a set of $n(n-1)$ simple paths specified for every ordered pair (x,y) of nodes. The routing will be feasible if an assignment of wavelengths to the paths can be given such that no link will carry in the same direction two different paths of the routing on the same wavelength. With such a routing, it is possible to perform gossiping in one round.

It is easy to see that $a'(G) \geq \aleph'(G) \geq \Delta(G)$ for any graph G. Here, $\aleph'(G)$ is the minimum number of colors used in any proper edge coloring of G, and is called the chromatic index of G.

2.HEXAGONAL NETWORK

Higher dimensional hexagonal graphs are the generalization of a triangular plane tessellation, and considered as a multiprocessor interconnection network. Nodes in a k-dimensional (k-D) hexagonal network are placed at the vertices of a k-D triangular tessellation, so that each node has upto $2k + 2$ neighbors.

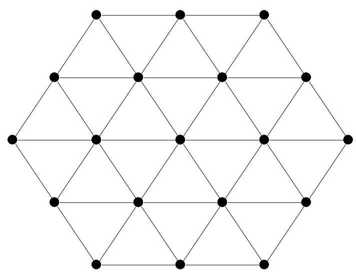


Figure 1: *HX*(2)

We call edges along a row as horizontal and the remaining edges as oblique acute and oblique obtuse edges.

Algorithm Acyclic edge-coloring *HX*(*n*)

Input: *HX*(*n*)

Algorithm:

1. Label alternate horizontal edges along row *i*, from left to right beginning from the first edge as 1 and the remaining edges as 2.
2. Label alternate oblique edges along column *i*, *i* odd, from left to right beginning from the first edge as 3 and the remaining edges as 4.
3. Label alternate oblique edges along column *i*, *i* even, from left to right beginning from the first edge as 5 and the remaining edges as 6.

Output: *HX*(*n*)= 6

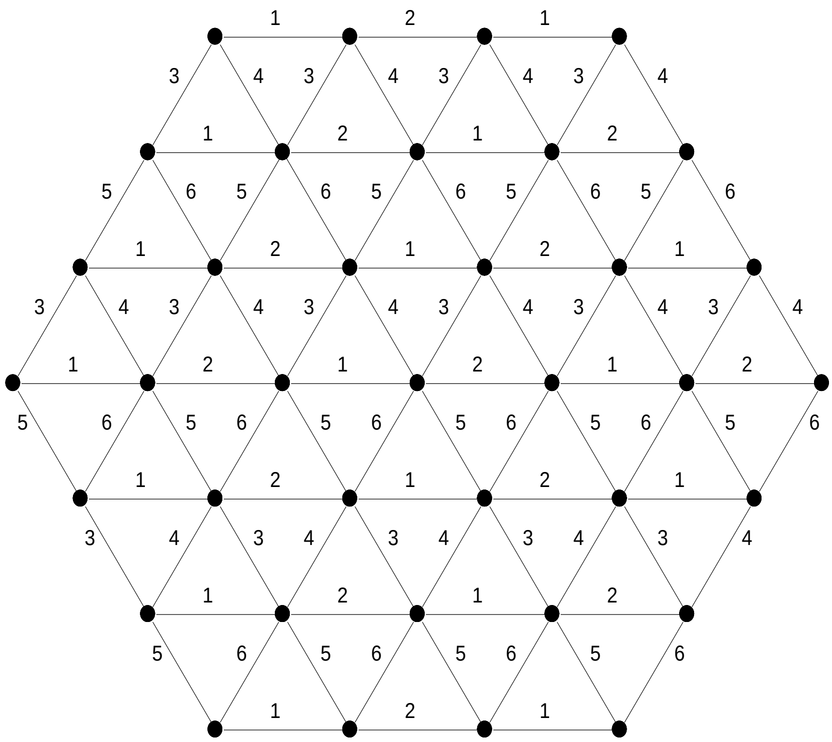


Figure 2: Acyclic Edge-coloring of *HX*(3)

Proof of Correctness:

Every cycle includes atleast one oblique acute, one oblique obtuse and one horizontal edge. By the algorithm these 3 colors are different. Thus the edges of any cycle are colored with at least three colors.

We have a′($HX(n)$) ≥ ∆($HX(n)$) = 6. By the algorithm, a′($HX(n)$) = 6. ■

3.HONEYCOMB NETWORK

Honeycomb networks are built recursively from hexagonal tessellation. The honeycomb network $HC(1)$ is a hexagon. The honeycomb network $HC(2)$ is obtained by adding six hexagons to the boundary edges of $HC(1)$. Inductively, honeycomb network $HC(n)$ is obtained from $HC(n – 1)$ by adding a layer of hexagons around the boundary of $HC(n – 1)$.

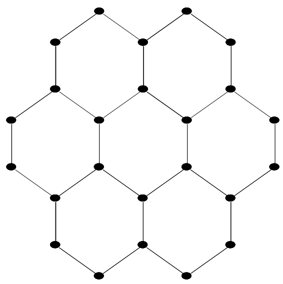


Figure 3*: HC*(2)

We call edges along a column as vertical edges and the remaining edges as oblique acute and oblique obtuse edges.

Algorithm Acyclic edge-coloring *HC*(*n*)

Input: Honeycomb network $HC(n)$

Algorithm:

1. Label all vertical edges along row *i*, as 1.
2. Label alternate oblique edges along row *i*, from left to right beginning from the first edge as 2 and the remaining oblique edges as 3.

Output: a′($HC(n)$) = 3

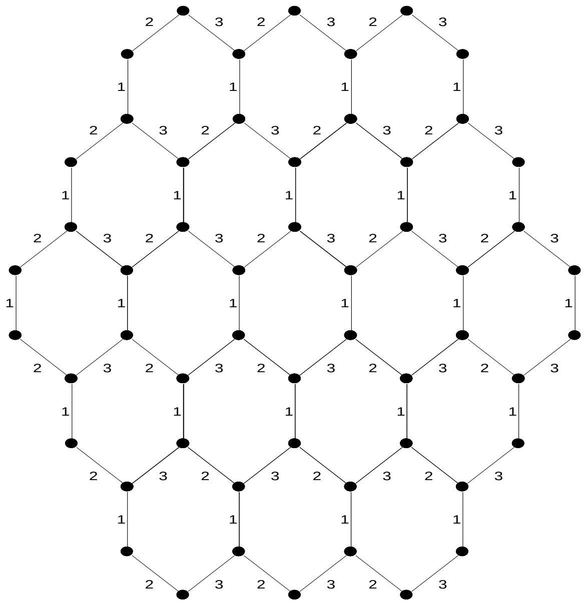


Figure 4: Acyclic Edge-coloring $HC(3)$

Proof of Correctness:

Every cycle includes atleast two oblique acute, two oblique obtuse and two vertical edges. By the algorithm these 3 colors are different. Thus the edges of any cycle are colored with at least three colors.

Since $a'(HC(n)) \geq \Delta(HC(n)) = 3$, we have $a'(HC(n)) = 3$. ∎

4.SIERPINSKI NETWORK

The Sierpinski triangle also called the Sierpinski gasket, is a fractal and attractive fixed set named after the Polish mathematician Wacław Sierpiński who described it in 1915. However, similar patterns appear already in the 13th century Cosmati mosaics in the cathedral of Anagni, Italy, and other places, such as in the nave of the Roman Basilica of Santa Maria in Cosmedin. Originally constructed as a curve, this is one of the basic examples of self-similar sets, i.e. it is a mathematically generated pattern that can be reproducible at any magnification or reduction.

Comparing the Sierpinski triangle or the Sierpinski carpet to equivalent repetitive tiling arrangements, it is evident that similar structures can be built into any rep-tile arrangements.

A fractal which can be constructed by a recursive procedure; at each step a triangle is divided into four new triangles, only three of which are kept for further iterations.

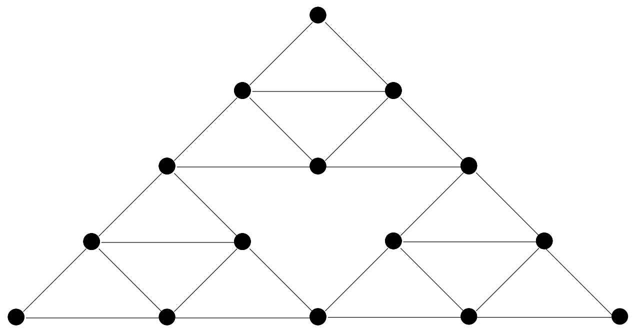


Figure 5: *S*(3)

Consider *S*(2). It contains three copies of *S*(1), Let us name the three copies of S(1) as Top, Left and Right. Name the edges as TL, TR, TB, LL, LR, LB, RL, RR, RB respectively. See figure 6.

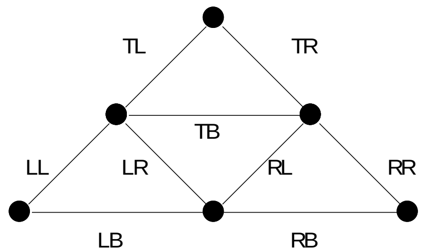


Figure 6

Algorithm Acyclic edge-coloring *S*(*n*)

Input: Sierpinski network *S*(*n*)

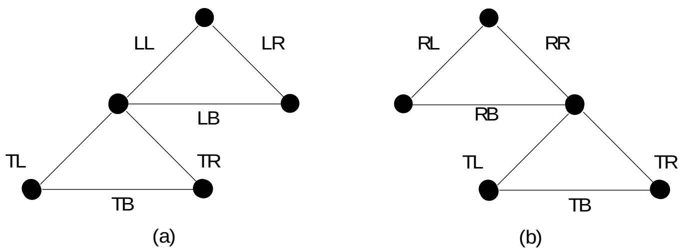
Algorithm:

Step 1: Arbitrarily color the edges of the top S(1), (ie) TL, TR, TB.

Step 2: TL=RR=LB; TR=LR; TB=RB

Step 3: LL=RL (Missing color of the four degree)

In order to know the color the edges of the top S(1) in the next level.



- **(i) Left:** LR=TL**;** LB=TB**;** TR=Missing color of the four degree. See figure (a).
- **(ii) Right:** RL=TR; RB=TB; TL =Missing color of the four degree. See figure (b).

Repeat: Step 1

Output: a′(S (n)) = 4

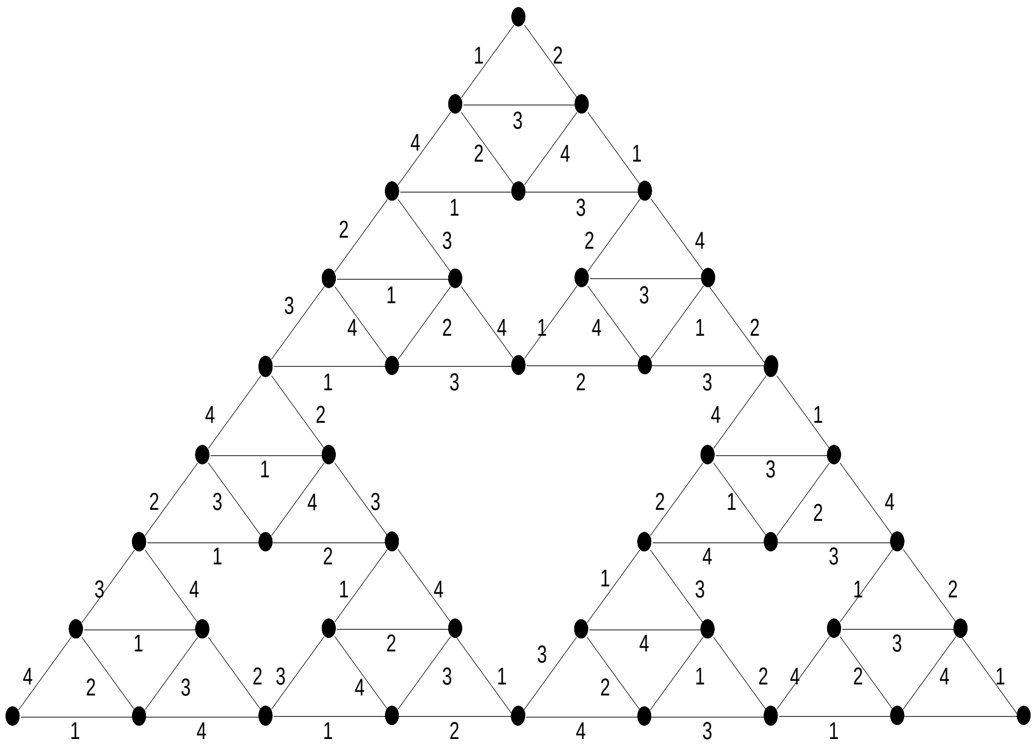


Figure 7: Acyclic Edge-coloring S(4)

Proof of Correctness:

Every cycle includes atleast one oblique acute, one oblique obtuse and one horizontal edge. By the algorithm these 3 colors are different. Thus the edges of any cycle are colored with at least three colors.

We have a′(S(n)) ≥ Δ(S(n)) = 4. By the algorithm, a′(S(n)) = 4. ■

5.CONCLUSION

The problem of acyclic edge-coloring for architecture such as butterfly, benes and pyramid networks are under investigation.

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