Fractal Image Compression Based On Complex Moments

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I. INTRODUCATION

Optimal fractal encoding consists of finding a finite set of contractive affine mappings one whose unique fixed point (attractor) is closest for the original image. Optimal fractal encoding makes fast encoding process with high quality of reconstructed image, therefore has been carried out to make the fractal image encoding technique by using fast Complex Moments (C.M_s) method . In this paper, we tried to provide a new technique in attempt to reduce the required computing steps which leads to speedup the encoding process by using C.M_s algorithm in a companion with fractal image compression.

Key word: fractal image compression, complex moment , fractal using complex moment

II. COMPLEX MOMENTS(CM$_S$)

Complex Moments (C.MS) are a vector of values, which represent the complex moment of an object around a proposed original. CMs are both very simple and quit powerful in providing an analytic characteristic for moment invariant. The computation of complex moment should involve with the calculation of its real and imaginary components [1][2].

The n-order complex moments for the image size n$\times$m calculated according to the following equation:

$$M_i = \sum_{x=1}^{n} \sum_{y=1}^{m} Z^i \mu \ (x, y)$$

Where,

i: indicates moments order, Z = (x+iy) is a complex function and μ (x, y) represents a pixel value at the position (x,y).

Complex moments have two parts, real and imaginary part, however, the computation of their values decompose into two directions: x-axis moment which represents real – part direction and y-axis moment for the imaginary- part direction (Z=x+iy). **Complex moments** may required a relatively long computation time and this time is increased with increase of moments order, because this will lead to increase of multiple additions and multiplication embedded within the calculation.

III. FRACTAL BASED ON COMPLEX MOMENT

From the particle view point of image processing the representation of image in terms of IFS parameters could represented a huge degree of data compression, compare, for example, the storage (uncompressed) of an image on a 1000 x 1000 pixel array, with gray – levels or color scales, to the possibility of representing this image by a few hundred IFS parameters [3][4].,therefore, for the affine transformations formations represented by

$[W_i$: P_i =1,2,3,…,N] = [(w_1,p_1),(w_2,p_2),(w_3,p_3),…,(w_n,p_n)

Where:

W_i the set of (N) affine transformation, P_i, is a set of probabilities.

It can be define the moments: $\overline{m}$ = (m_1,m_2, …m_n) as flows

$$M(\mu, j) = \int_k z^j \, d\mu(z) \qquad \forall j, \ldots\ldots(1)$$

Bransley and Demeko [5] showed that these moments can be calculated, uniquely, explicitly recursively in term of the lower order moment and the parameters which define the IFS, i.e.

$$M(\mu \quad m)= \sum_{\ell=0}^{m-1} C_{\ell}^{m} \sum_{i=1}^{d} p_i t_i^{m-\ell} s_i^{\ell} M (\mu, \ell) / [1-\sum_{i=1}^{d} p_i s_i^{m}]. \quad ..(2)$$

Where,

$$C_{\ell}^{m} = \frac{m!}{\ell!(m-\ell)!}$$ (is the binomial coefficient)

And

$t_i = (1-s_i)\, a_i$

Where:

$|s_i|<1$ is called the magnification parameter and (a_i) is called the center point (complex number), thus, it is expected that the difference between moments measured directly from the target image by using eq. (1) and the corresponding values calculated directly from eq. (2) is a good measure for the degree of closeness between the IFS- code and the target set.

IV. FRACTAL IMAGE COMPRESSION BASED ON COMPLEX MOMENT

Fractal image compression using Complex Moments is new method applied in this filed to speeding up the encoding process. This method based on uniform partitioning image (fixed size squares block). Speeding comes from low number of computation process.

V. COMPLEX MOMENT ENCODING PROCESS

The basic idea of CM_s method based on utilizing new matching criteria. This criterion involves the utilization of the moment domain and range blocks in computing the offset – scale transform coefficient values. To driven the mathematical based of using CM_s based on fractal image compression it can be start from the affine transformation and define the moments, as flows:

Where,

$$M_{(m,n)} = \int f(z^n) d\mu ....(3)$$

M: moments and f (z) complex function when z=x+iy

$$f(z) = Z^n R. \;.........(4)$$

Eq.(3) become

$$M = \int z^n R \, d\mu ....(5)$$

Using the discrete form of eq. (5)

$$M = \sum z^n R ....(6)$$

Appling affine transformation

$$R \approx sD + o \;\text{-----}(7)$$

Appling eq. (7), on the eq. (6)

$$M_r^{(n)} = \sum Z^n (sD+o) = s\sum Z^n D + oZ \;............(8)$$

Where,

M_r: Range moments

Z^n : order of moments in this work , the order ranging from (1→ 6)

$$\frac{\partial^2 x}{\partial s^2} = M_D^{(n)} M_y^{(n)} = s\sum M_{D^{(n)\,2}} + o\sum M_D^{(n)} W_{(n)}. = 0....(9)$$

$$\frac{\partial^2 x}{\partial o^2} = \sum M_r^{(n)} W_n = s\sum M_D^{(n)} W_{(n)} + o\sum W_{(n)}^2 = 0.............(10)$$

By using the least square method to find the scale and offset coefficients, must be applied the condition of least square method: (11&12)

by solving eqs. (9& 10) to find (s & o)

From eqs. (11 &12) compute

$$E(R,D) = \sum_{x=0}^{255} M_R (-s M_D - o W_n)^2 \qquad (13)$$

Complex Moment method is straightforward to apply to gray level scale image. Image is partitioned into blocks of fixed size; 4×4 pixels (i.e. Jacqua'n partitioning) eqs. (9 &10) have been used to determine the affine transform coefficients (i.e. s and o). Also, the type of complex moments (1-4) determine to compute the domain block and the order of moments (1-6) have been utilize to evaluate the degree of fitness between the range and domain (R-D) couples. The encoding process starts with compute the complex moment for each domain block for given its type and order. After that store the optimal domain block, which satisfy eqs. (11 &12) in the codebook. From eq. (13) find the block D_k with minimal error (i.e. $E(R,D_k)$,< $E(R,D_i)$ and store the complex moment domain block address.

VI. ENCODING PROCESS COMPLEXMOMENT ALGORITHM

Encoding process achieved by these steps:

1- Partition the image using classical partition method (fixed size squares block).
2- Construct domain pool to contain moment domain block.
3- For each domain block calculate the moment the same type (odd or even) of all image.
4- Store all order of moments in a domain pool.
5- Start encoding process for each range block.

a) Initialize matching list set counter number = 0.

b) Then point to the first range block in the image (up-left-corner).

c) Calculate the moment of the range block (by determined the number of moment).

d) Calculate the (s & o) coefficients of the same range block size.

e) Use the coefficients (s & o) to compute the error E (R, D).

f) Among domain pool find the block D_k with minimal error

$$E(R, D_k) < E(R, D_i)). \qquad \forall i \neq k .$$

g) Store the moment domain block address in the matching list, and increment the counter number of matching list by one.

h) Form type and order of complex moments of domain block list in the "matching list" apply the fractal matching using the traditional method that discussed in this chapter.

6-Repeat step (5) until no rang block is existed.

VII. RESULTS AND DISCUSSION

The results of $C.M_s$ method need to be observed two closely related quantities. First one is the compression ratio and the second quantity measures of quality (PSNR). It is an open problem defining the visual qualities of an image approximation in a mathematically expressible way. In order to compare C.Ms method with Jacquin's method , Jacquin's encoder with fixed size block (4×4) pixels is built. Then apply C.Ms method using Jacquin's encoder . also, the encoding algorithm apply with fixed size block (4×4) pixels, after that apply full search of matching to encode the image. Table (1) showing the relationship between all number of Complex Moment and PSNR, with two types of C.Ms (odd and even). The performances of both methods are comparable table (2) show as the results of two methods (Jacquan and C.Ms). From the results that's utilize in this table high values of PSNR in Jacquan's method, it means high quality of reconstructed image, also saver long encoding time in Jacquan's method Vic versa C.Ms method have fast encoding time, while preserving C.R and PSNR values to those obtained when conventional full search encoding with Jacquan's method as show in figs. (1&2). These figs. showing the best order of C.Ms with two types (odd and even) , whose choice to apply in encoding process by test image (i.e., bird image). Finely, The reconstructed image has high quality and speeding up the encoding time when we applied C.Ms method. The aim of this work.

Type of Complex Moment							
Odd=1		Odd=3		Even=2		Even=4	
No. Of orders C.Ms	PSNR (dB)	NO.of Order C.Ms	PSNR (dB)	No. Of orders C.Ms	PSNR (dB)	No. Of orders C.Ms	PSNR (dB)
1	27.32	1	27.57	1	27.55	1	27.13
2	28.73	2	28.90	2	29.13	2	28.62
2	29.02	3	28.55	3	28.87	3	28.97
4	29.16	4	28.38	4	28.75	4	29.23
5	29.62	5	28.54	5	28.90	5	29.01
6	29.51	6	27.57	6	28.75	6	29.12

Table(1): The relationship between all orders of C.Ms of two type(odd,even) and PSNR applied on the bird image of size (256 $\times$256) pixels.
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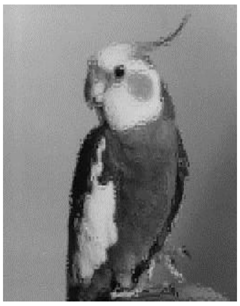
Complex moments method						Jacqu'an method			
Range block size	Type of C.Ms	No. Of order C.Ms	PSNR (dB)	C R	T. sec	Range block size	PSNR (dB)	CR	T. Sec
4	Odd =1	5	29.628	4	90	4	30.43	3.5	1060
4	Odd=3	2	28.903	4	73				
4	Even= 2	2	28.131	4	68				
4	Even= 4	4	29.231	4	82				

Table (2): The encoding results of classical partition applied on the bird image of size (256 $\times$256)

Original image

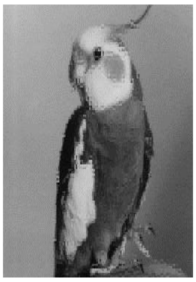
PSNR = 29.62
C.R= 4
Time = 90 sec
Type of moment =1
Complex



PSNR = 28.90
C.R= 4
Time = 73
Type of complex moment =3
Complex Moment = 2

Fig (1): The result of complex moment methods Applied on bird image when odd type of complex moment.

PSNR = 28.13
C.R= 4
Time =68 sec
Type of Complex Moment = 2
Complex Moment = 2



PSNR = 29.231
C.R= 4
Time =68 sec
Type of complex moment = 4
Complex Moment = 4

Fig (2): The result of complex moment methods Applied on bared image when even type of moment

VIII. RECONSTRUCTION IMAGE

Fig. (3) shows the relation between PSNR and iteration applied on bird image and fig. (4) show detail added at each iteration of the fractal compression image result. Illustrated the reconstructed image can be started from black image (initial) until reached the attractor at 8th iteration. Also, it presents the reconstructed image by using the suggestion block size of (4x4) pixels.

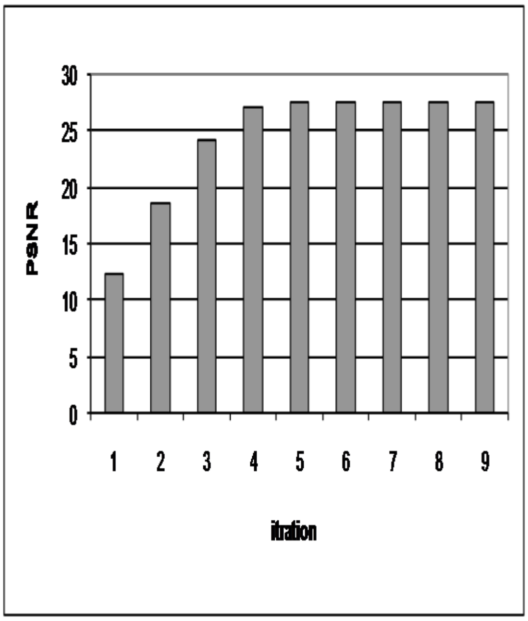


Fig. (3):The effect of increasing number of iteration to reconstruct the image

Original image

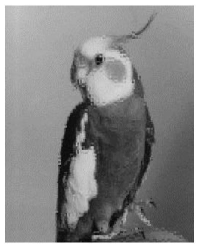
a)

b

c

d

e



f

Fig. (4): The reconstruction (decoding) process of bird image (a) The initial (Empty) image (b) the 1st iteration (c) the 2nd iteration (d) the 3rd iteration (e) the 5th iteration, and (f) the 6th iteration .

IX . CONCLUSION

The main purpose behind the C.Ms method is to speed up the slow classical encoding process. Thus it is implemented & applied on the bird image, and the following remarks are noticed.

1-This method is fast in contrast with the traditional, Jacquan's method, due to C.Ms method utilize rather than affine transform.

2-In a case of a high order of the complex moments in the image become most clear, and that is remarked throughout the PSNR in contrast with the complex moments, due to the extra computing process.

3-The changes in the orders of complex moments is only to apply fitness between (R & D) couple

4- It is noticed throughout the encoding process that the search of the complex moments domain is done in the domain pool which in satisfy the $E(R,D_i) > E(R,D_k)$ fact and the least value is considered . The least value is considered and inserted in the codebook with the position, and the search process will go on until the arrival of the last range block in the image. The less test error carried value is always stored.

6-Both (Jacquan's and C.Ms) methods are gained high quality (i.e., high values of PSNR). So C.Ms method is suit for speeding up the encoding time, and the reconstructed image clearness.

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