Nonlinear Time Structures for Economic Reality Description – a Survey

Maria Antonina Mach
University of Economics, Wroclaw, Poland
maria.mach@ue.wroc.pl

***Abstract* - In the paper the nonlinear time structures are presented. They are discussed, and their formalization is shown. Next, I present some sample economic situations which can be formally described using these nonlinear structures. Also the possibility of combining different nonlinear time structures to describe complex economic phenomena is discussed.**

***Keywords* – nonlinear time, economic environment**.

Introduction

The environment of modern enterprises is used to be called a turbulent one, which means, among others, that it is heterogeneous and highly changeable. It is therefore very difficult to capture changes in the environment, to point out the meaningful ones, to find their causes, effects and directions, and to see their influence on enterprise's activity. At the same time, these changes influence enterprises' well-being and development in a very serious manner.

The analysis of changes has to be linked with the temporal aspect, because time is strictly connected with the notion of change. The temporal aspect is important also because in modern enterprises knowledge is the more and more crucial. And mostly knowledge has temporal characteristics; therefore omitting the temporal aspect would lead to omitting the important elements of knowledge. In this way, time becomes a very important category for any enterprise.

In the context of what was said above, it must be pointed out that the complete knowledge representation is a temporal one. As Sowa claims [14], knowledge representation consists of three elements: ontology, logic, and computational procedures. Each logic assumes ontology of the domain being depicted. Therefore the representation of knowledge about the dynamic environment of an enterprise should consist of time and domain ontologies, of temporal logic and of computational procedures (reasoning). If one assumes that time structure influences the choice of temporal entities, it becomes clear, that time structure is absolutely fundamental.

In the paper I deal with nonlinear time structures and their applicability to represent different economic situations. Some authors dealing with the question of such time structures, are – among others – Bennett [2], Zanardo [16], Emerson and Halpern [3], or Glabbeek [4].

In the paper I assume the first order predicate calculus, I do not discuss time structures for temporal modal logics, although they are considered a historical source of temporal formalization, because the first temporal logic was the modal tense logic by Prior [12].

I. Selected simple nonlinear time structures

The beginnings of time analysis can be found in philosophical works, in which it was discussed, what is time at all. Let us cite, for example such works as [13] or [11]. Nevertheless I will not deal with this question here. What is important for us is how time structure is linked with artificial intelligence, or more precisely, with knowledge representation with the use of temporal logic.

The first question of structure concerns time density or discreteness. The discrete time is treated as a set of some elements, while the dense one – as such a set, that between any two elements there is always a third element. In other words, the discrete time is modeled as a discrete set which can be perceived as a set of integers, while the dense time is modeled as a continuum – perceived as a set of reals [15], [8]. Formally, the time structure T is called dense, if:

$$\forall t_1, t_2 \in T\ (\exists t_3 \in T: t_1 < t_3 \wedge t_3 < t_2),$$

while time structure T is called discrete, if for any $t_1, t_2 \in T$:

- $t_1 < t_2 \Rightarrow \exists t_3: (t_1 < t_3) \wedge \neg(\exists t_4: t_1 < t_4 \wedge t_4 < t_3)$;
- $t_2 < t_1 \Rightarrow \exists t_3: (t_3 < t_1) \wedge \neg(\exists t_4: t_3 < t_4 \wedge t_4 < t_1)$.

The next problem concerns the question, whether time is bounded or unbounded: it may be unfinished in one or both directions with the respect to a certain point, called "now".

And finally, linearity versus nonlinearity of time. The most widely assumed model of time is the linear one, which can be graphically depicted as a straight line. Formally, a time structure T is called a linear one, if ([7], p. 20):

$$\forall t_1, t_2 \in T: (t_1 < t_2) \vee (t_1 = t_2) \vee (t_2 < t_1).$$

Fig. 1. Linear time. Source: own elaboration.

The non linear time models are as follows: time branching into the future, time branching into the past, time branching in both directions (that is parallel one) and cyclic time. A motivation for branching time structures was that many different pasts ("ways") could have led to present moment, and many different "ways" can lead from "now" to the future. The formal definitions are as follows [7, p. 21nn]:
a time structure T is called branching into the future, that is a left-linear one, if

$$\forall t_1, t_2, t_3 \in T\ (t_2 < t_1 \wedge t_3 < t_1) \Rightarrow (t_2 < t_3) \vee (t_2 = t_3) \vee (t_3 < t_2).$$

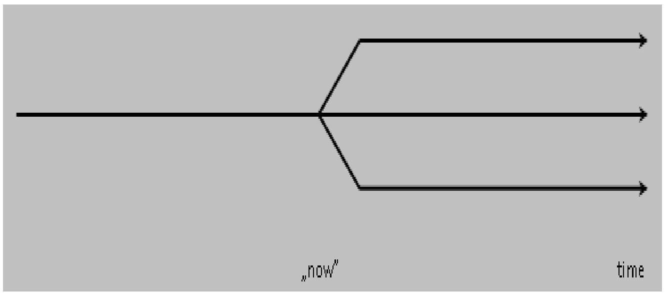

Fig. 2. Time branched into the future.

Source: own elaboration.

A time structure T is called branching into the past, that is a right-linear one, if

$$\forall t_1, t_2, t_3 \in T\ (t_1 < t_2 \wedge t_1 < t_3) \Rightarrow (t_2 < t_3) \vee (t_2 = t_3) \vee (t_3 < t_2).$$

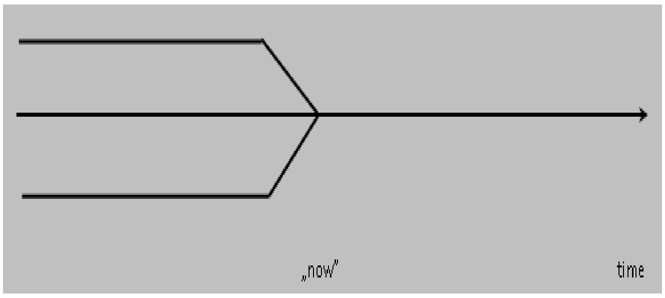

Fig. 3. Time branched in the past.

Source: own elaboration.

A time structure T is called a parallel one, if it is both left- and right-linear one, that is branching in both directions.

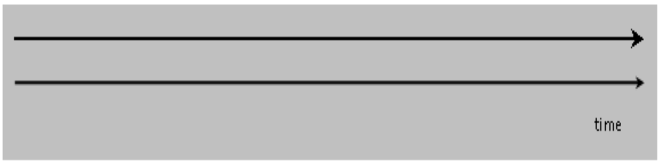

Fig. 4. Parallel time.

Source: own elaboration.

A cyclic time structure is rarely discussed in the literature, but it is worthy mentioning here. A metric point cyclic time structure T is an ordered tuple $\langle T, C, <, <^*, \delta, S \rangle$, where: T – set of time points, C – set of distances between points, $<$ - global ordering over T, $<^*$ - local ordering T, δ – metric over T, S – longitude of semicircle.

For each point $x \in T$ there exists exactly one point $x^* \in T$ such, that $\delta(x, x^*) = S$. These two points divide the circle into two semicircles. The cyclic time structure is characterized by [5, p. 30]:

- completeness: $\forall x, y\ (x < y)$,
- local antisymmetry: $\forall x, y\ (x <^* y \rightarrow \neg(y <^* x))$,
- local linearity: $\forall x, y\ ((x \neq y\ \&\ x \neq y^*) \rightarrow (x <^* y \vee y <^* x))$,
- local transitivity: $\forall x, y, z\ ((\delta(x, y) + \delta(y, z) < S) \rightarrow (x <^* y\ \&\ y <^* z \rightarrow x <^* z))$,
- consistency: $\forall x, y, z\ ((\delta(x, y) + \delta(y, z) < S) \rightarrow (x <^* y\ \&\ y <^* z \rightarrow \delta(x, y) + \delta(y, z) = \delta(x, z)))$.

Up till now the research on temporal reasoning has dealt mostly with linear time models, the use of which in economy and management was described e.g. in [9]. The next parts of the paper will be devoted to nonlinear time structures.

II. Sample economic situations and time structures to depict them

In the introduction I restricted myself to structures connected with the first order predicate calculus. Here I should make one more restriction, concerning basic temporal entities. Generally speaking, in time ontology one may assume time points, time intervals or both of them as basic entities (see e.g. [Vila 1994]). In the paper I assume time composed of points. The reason for this assumption is the domain for which time structures will be used – namely the economic domain. Although many elements in the economic environment of an enterprise change in a continuous manner, and some authors claim that this environment should be monitored in the same way [10], there are equally many elements that change discretely. Moreover, from the practical point of view, it is impossible to "feed" the intelligent system with information continuously. Changes have to be captured discretely. And finally, if one assumes a continuous time, one should also assume second order axiomatization [1, p. 36], while in the paper I assume – as it was already said – a first order one.

As Kania claims [6, p. 62], establishing time structure is necessary to properly perform operations on values defining time, to operate temporal knowledge base etc. First of all, the time structure has to be adequate to the economic phenomenon being depicted to enable modeling it. Let me illustrate this using a few examples.

Example 1.
An enterprise X has been established by the merger of two other enterprises. Therefore its present situation and strategy is partly dependent on the history of these previously operating enterprises, for example because of business convictions of their CEO's. A good time model to depict this situation is time branching into the past (right-linear one), where two axis of enterprises' histories converge in a moment (time point) of merger, and from this time point there is only one time axis – one history of the enterprise.

Formally I denote this situation as follows. I assume a point time structure $\mathcal{T}$, being an ordered pair $\langle T, <\rangle$, where T – a set of time points, while < - precedence relation over T. This structure is:

– transitive: $\forall x, y\ (x < y \ \&\ y < z \rightarrow x < z)$,
– antireflexive: $\forall x\ \neg(x < x)$.

Recall from Section 2, that generally right linearity is denoted as:

$$\forall t_1, t_2, t_3 \in T\ (t_1 < t_2 \wedge t_1 < t_3) \Rightarrow (t_2 < t_3) \vee (t_2 = t_3) \vee (t_3 < t_2),$$

while in this concrete example one could write e.g.:

- $(t_1, t_2, t_3 < t_F) \Rightarrow (\forall t_1, t_2, t_3 \in T\!: (t_1 < t_2 \wedge t_1 < t_3) \Rightarrow (t_2 < t_3) \vee (t_2 = t_3) \vee (t_3 < t_2))$,
- $(t_1, t_2 > t_F) \Rightarrow (\forall t_1, t_2 \in T\!: (t_1 < t_2) \vee (t_1 = t_2) \vee (t_2 < t_1))$,

where t_F denotes the day of enterprises' merger.

Example 2.
The left-linear structure – that is time branching into the future – is somehow a mirror structure to the previous one. It has the same features of transitivity and antireflexitivity, and is generally denoted as (see Section 1):

$$\forall t_1, t_2, t_3 \in T\ (t_2 < t_1 \wedge t_3 < t_1) \Rightarrow (t_2 < t_3) \vee (t_2 = t_3) \vee (t_3 < t_2).$$

An economic situation that could be depicted using this time structure is for example an analysis of different variants of market strategy. An enterprise takes into account different pace and scope of actions to be taken, which can bring different effects. Therefore, from the point called "now", the development of an enterprise may take different "ways" depending of market strategies. Using the structure of time branching into the future, I can denote this as follows:

- $(t_1, t_2 < t_S) \Rightarrow (\forall t_1, t_2 \in T\!: (t_1 < t_2) \vee (t_1 = t_2) \vee (t_2 < t_1))$,
- $(t_1, t_2, t_3 > t_S) \Rightarrow (\forall t_1, t_2, t_3 \in T\ (t_2 < t_1 \wedge t_3 < t_1) \Rightarrow (t_2 < t_3) \vee (t_2 = t_3) \vee (t_3 < t_2))$,

where t_S is a boundary, conventional point between linear and branching time, denoting the moment of taking a certain strategy.

Example 3.
Consider now one enterprise, but operating on different markets, let's say geographical ones. Depending on the conditions on these markets, the pace of enterprise's development, its strategies etc. may differ. Therefore if one puts all the actions of this enterprise on every market on the same time axis (the linear time structure), this would be a simplification going much too far. It seems that an adequate structure in this case would be rather a parallel one. As Klimek points out, such a structure contains time points that can be impossible to compare with each other because of the precedence relation [7, p. 22]. Using such a structure is the more justified in this example, as the points situated one "over" another on different time axis do not correlate in calendar terms.

Example 4.
The next classical time structure is a cyclic one. It is worthy mentioning that it is the only structure among these discussed in the previous Section, which has a metric defined. This characteristics, along with other features, make the cyclic structure useful for description of such economic phenomena, as business cycles, stock market cycles etc.

The question of reasoning about economic phenomena, described by different time structures, is left opened. It is beyond the scope of this paper. As it has been pointed out in the Introduction, the representation of any domain is composed of: ontology, logic and computational procedures (reasoning). The reasoning procedures result from the logical formalism adopted. An excellent survey of reasoning algorithms for nonlinear time structures based on first order predicate calculus may be found in [5].

III. Composed nonlinear time structures and their use in formalizing economic situations

In the previous Section I have shown some examples of using classic nonlinear time structures to formalize economic situations. One may nevertheless imagine such situations, when these classic structures would not be sufficient. Consider for example a situation, when two enterprises merge – as in the example 1 in Section 2, then for some time operate as a single enterprise, and then divide again into two firms, but closely cooperating, so it is purposeful to analyze their development together, but not on the same time axis. In this case I deal with the following time structures: right-linear one, linear, and left-linear one, respectfully. Formally this situation might be written as follows:

- $(t_1, t_2, t_3 < t_F) \Rightarrow (\forall t_1, t_2, t_3 \in T: (t_1 < t_2 \wedge t_1 < t_3) \Rightarrow (t_2 < t_3) \vee (t_2 = t_3) \vee (t_3 < t_2))$,
- $((t_1, t_2 > t_F) \wedge (t_1, t_2 < t_P)) \Rightarrow (\forall t_1, t_2 \in T: (t_1 < t_2) \vee (t_1 = t_2) \vee (t_2 < t_1))$,
- $(t_1, t_2, t_3 > t_P) \Rightarrow (\forall t_1, t_2, t_3 \in T\ (t_2 < t_1 \wedge t_3 < t_1) \Rightarrow (t_2 < t_3) \vee (t_2 = t_3) \vee (t_3 < t_2))$,

where: t_F – the date of merger, t_P – the date when the firm divides.

Such a structure is:

– transitive: $\forall x, y\ (x < y \;\&\; y < z \rightarrow x < z)$,
– antireflexive: $\forall x\ \neg(x < x)$,
– antisymmetric: $\forall x, y\ (x < y \rightarrow \neg(x < y))$,
– discrete: $\forall x, y\ (x < y \rightarrow \exists z(x < z \;\&\; \neg\exists u(x < u \;\&\; u < z)))$
$\forall x, y\ (x < y \rightarrow \exists z(z < y \;\&\; \neg\exists u(u < u \;\&\; u < y)))$.

Obviously, depending on particular needs, one may merge different time structures into different configurations. It seems that the easiest and the most justified are merging the structures of branching and linear time. There can be many combinations of structures similar to the one shown above; one can imagine for example a chain composed of branching and linear structures. On the other hand it seems difficult, if not impossible at all, to merge the cyclic time structure with branching and/or linear ones, because the cyclic structure is a closed one.

IV. Possible practical implementations of nonlinear time structures in the economic domain

In [9] a temporal intelligent system for the analysis of changes in enterprises' environment has been proposed. It is an artificial intelligence system, which perform temporal reasoning in an explicit way. Therefore it contains not only a fact base, a rule base and an inference engine, but also deals with the question of time explicitly. It allows for reasoning about changes of phenomena in time, for historical analysis of phenomena, for analysis of future changes – generally speaking it is capable of dynamic analysis of reality. This system assumes linear structure of time. One may nevertheless think of a temporal intelligent system, based on temporal first order formalisms, but assuming one of nonlinear time structures – depending on system's tasks or on the system's domain (see examples in Sections 2 and 3). Apart from the temporal analysis of economic environment, such a system, thanks to the use of nonlinear time structures, might be useful for the following tasks:

- analysis of business cycles (cyclic time structure),
- analysis of firm's history and trend development prediction (branching structures),
- analysis of different strategy variants (left-linear structure),
- analysis of firm's development on different markets in the same time period (parallel time structure).

The use of a temporal intelligent system may be of profit to an enterprise, resulting from the advantages of temporal formalization and non-classical time structures. These profits encompass:

- the possibility of using fuzzy or unsure knowledge,
- the possibility of capturing the "relativity" of knowledge (e.g. A happened before B), which very often has no reference to dates,
- possibility of using different inference granulations,
- persistence modeling.

These are the advantages resulting from the power of temporal formalisms (see [9, p. 63-70] for details). Thanks to them, an enterprise is able to make a better use of knowledge coming from its environment than if time were omitted by information systems.

Conclusions

Modern enterprises that operate in a turbulent environment need new information systems tools that would aid decision makers in the strategy process, allowing them to fully and quickly react to changes in this environment.

The basic step for building such a tool (system) is a proper representation of temporal knowledge, and one of the major ontological questions to be answered s the structure of time. The most often the linear structure is assumed. In many cases, however, this structure is insufficient, because there are economic situations for which this structure is not applicable.

In the paper I presented some selected nonlinear time structures along with a few examples of economic situations, which could be formalized with the use of these structures. Possible practical applications of the structures have also been pointed out, namely their implementation in a temporal intelligent system. Such a system would serve as a support for decision-making analysis; it could also serve as a so-called competitive intelligence system. It could also become one of the elements of information infrastructure in the enterprise, together with such solutions, as data warehouses, knowledge management systems and other tools.

Bibliography

[1] Bennett B., Galton A. P. (2004), *A unifying semantics for time and events*. “Artificial Intelligence”, Vol. 153, pp. 13-48.

[2] Bennett B. (2003), *A Logic of Branching Histories with a Shared Linear Time Series*. Foundations and Applications of Spatio-Temporal Reasoning (FASTR): Papers from the 2003 Symposium, ed. H. W. Guesgen, D. Mitra, J. Renz. AAAI, Menlo Park, CA.

[3] Emerson E. A., Halpern J. Y. (1986), *“Sometimes” and “Not Never” Revisited: On Branching versus Linear Time Temporal Logic*. “Journal of the Association for Computing Machinery” Vol. 33, No. 1, pp. 151-178.

[4] Glabbeek van, R. J. (2001), *What is branching time semantics and why to use it?* In: “Current trends in theoretical computer science: entering the 21st century”. World Scientific Publishing CO., Inc., River Edge, NJ, USA, pp. 469-479.

[5] Hajnicz E. (1996), *Time Structures. Formal Description and Algorithmic Representation.* Lecture Notes in Artificial Intelligence 1047, Springer, Berlin.

[6] Kania K. (2004), *Temporalne bazy danych w systemach informatycznych zarządzania*. Prace Naukowe Akademii Ekonomicznej im. Karola Adamieckiego w Katowicach, Wydawnictwo AE Katowice.

[7] Klimek R. (1999), *Wprowadzenie do logiki temporalnej*. Uczelniane Wydawnictwa Naukowo-Dydaktyczne AGH, Kraków.

[8] Knight B., Ma J. (1994), *Time representation: A taxonomy of temporal models*. „Artificial Intelligence Review”, Vol. & No. 6, Kluwer Academic Publishers.

[9] Mach M. A. (2007), *Temporalna analiza otoczenia przedsiębiorstwa. Techniki i narzędzia inteligentne.* Prace Naukowe AE Wrocław nr 1154, seria „Monografie i Opracowania” nr 176, Wrocław.

[10] Przybyłowski K., Hartley S. W., Kerin R. A., Rudelius W. (1998), *Marketing. Pierwsza polska edycja*. Dom Wydawniczy ABC Sp. z o. o.

[11] Prior A. N. (1955), *Diodoram modalities*. “Philosophical Quaterly”, Vol. 5, pp. 205-213.

[12] Prior A. N. (1967), *Past, present and future*. Clarendon Press, Oxford.

[13] Russell B. (1913), *On the notion of cause*. “Journal of the Aristotelian Society”, Vol. 13, pp. 1-26.

[14] Sowa J. F. (2000), *Knowledge representation*. Brooks/Cole, Pacific Grove, CA.

[15] Vila L. (1994), *A Survey on Temporal Reasoning in Artificial Intelligence*. AI Communications, 7(1), pp. 4-28

[16] Zanardo A. (2002), *First-Order and Second-Order Aspects of Branching-Time Semantics*. Proc. HPLMC-02: Second International Workshop on the History and Philosophy of Logic, Mathemathics and Computation; Donostia – San Sebastian, Spain, 7-9 November.