A Model for System Reliability Optimization Problems Based on Ant colony Using Index of Criticality Constrain

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ABSTRACT

This paper presents a new meta-heuristic-based model for complex reliability problems. A model maximizes the system reliability and minimizes the cost. The model gives procedures to enhance engineers' ability to design system for which reliability is an important consideration. It involves only the reliability of components as variables. In addition, the model takes in to its consideration, the criticality of system components as constrains. The components are characterized by their cost and criticality. The model solved by adapting ant colony optimization (ACO) method to deal with the highly constrained problem. The algorithm has been thoroughly tested on bench mark problems from the literature. Our numerical experiences show that our approach is promising especially for complex systems. The proposed model proves to be robust with respect to its parameters.

Key Words: System reliability, Complex system, Ant colony, Component's criticality

1. Introduction.

System reliability can be defined as the probability that a system will perform its intended function for a specified period of time under stated conditions [1]. Many modern systems, both hardware and software, are characterized by a high degree of complexity. To enhance the reliability of such systems, it is vital to define techniques and models aimed at optimizing the design of the system itself. This paper presents a new metaheuristic-based algorithm aimed at tackling the general system reliability problem, where one wants to identify the system configuration that maximizes the overall system reliability, while taking into account a set of resource constraints. Estimating system reliability is an important and challenging problem for system engineers. [2].

It is also challenging since current estimation techniques require a high level of background in system reliability analysis, and thus familiarity with the system. Traditionally, engineers estimate reliability by understanding how the different components in a system interact to guarantee system success. Typically, based on this understanding, a graphical model (usually in the form of a fault tree, a reliability block diagram or a network graph) is used to represent how component interaction affects system functioning. Once the graphical model is obtained, different analysis methods [3–5] (minimal cut sets, minimal path sets, Boolean truth tables, etc.)

can be used to quantitatively represent system reliability. Finally, the reliability characteristics of the components in the system are introduced into the mathematical representation in order to obtain a system-level reliability estimate. This traditional perspective aims to provide accurate predictions about the system reliability using historical or test data. This approach is valid whenever the system success or failure behavior is well understood. In their paper, Yinong Chen, Zhongshi He, Yufang Tian [6]. They classified system reliability in to topological and flow reliability. They considered generally that the system consists of a set of computing nodes and a set of components between nodes. They assume that components are reliable while nodes may fail with certain probability, but in this paper we will consider components subject to failure in a topological reliability.

Ideally, one would like to generate system design algorithms that take as input the characteristics of system components as well as system criteria, and produce as output an optimal system design, this is known as system synthesis[7], and it is very difficult to achieve. Instead, we consider a system that is already designed then try to improve this design by maximizing the components reliability which will maximize the over all system reliability. In the most theoretical reliability problems the two basic methods of improving the reliability of systems are improving the reliability of each component or adding redundant components [8]. Of course, the second method is more expensive than the first. Our paper considers the first method. The aim of this paper is to obtain the optimal system reliability design with the following constrains. :

1: Basic cost reliability data to components; we use a linear-cost-reliability relation for each component [7].

2: Criticality of components [9]. The designer should take this in to account before building a reliable system and according to criticality of component increasing reliabilities will go toward the most critical component. Components' criticality can be derived from its failure effects to system reliability failure. Which the position of a component will play an important role for it's criticality which we called it the index of criticality.

2. System reliability problem

2.1 Literature view

Many methods have been reported to improve system reliability. Tillman, Hwang, and Kuo [10] provide survey of optimal system reliability. They divided optimal system reliability models into series, parallel, series-parallel, parallel-series, standby, and complex classes. They also categorized optimization methods into integer programming, dynamic programming, linear programming, geometric programming, generalized Lagrangian functions, and heuristic approaches. The authors concluded that many algorithms have been proposed but only a few have been demonstrated to be effective when applied to large-scale nonlinear programming problems. Also, none has proven to be generally superior. Fyffe, Hines, and Lee [11] provide a dynamic programming algorithm for solving the system reliability allocation problem. As the number of constraints in a given reliability problem increases, the computation required for solving the problem increases exponentially. In order to overcome these computational difficulties, the authors introduce the Lagrange multiplier to reduce the dimensionality of the problem. To illustrate their computational procedure, the authors use a hypothetical system reliability allocation problem, which consists of fourteen functional units connected in series. While their formulation provides a selection of components, the search space is restricted to consider only solutions where the same component type is used in parallel. Nakagawa and Miyazaki

[12] proposed a more efficient algorithm. In their algorithm, the authors use surrogate constraints obtained by combining multiple constraints into one constraint. In order to demonstrate the efficiency of their algorithm, they also solve 33 variations of the Fyffe problem. Of the 33 problems, their algorithm produces optimal solutions for 30 of them. Misra and Sharma [13] presented a simple and efficient technique for solving integer-programming problems such as the system reliability design problem. The algorithm is based on function evaluations and a search limited to the boundary of resources.

In the nonlinear programming approach, Hwang, Tillman and Kuo [14] use the generalized Lagrangian function method and the generalized reduced gradient method to solve nonlinear optimization problems for reliability of a complex system. They first maximize complex-system reliability with a tangent cost-function and then minimize the cost with a minimum system reliability. The same authors also present a mixed integer programming approach to solve the reliability problem [15]. They maximize the system reliability as a function of component reliability level and the number of components at each stage. Using a genetic algorithm (GA) approach, Coit and Smith [16], [17], [18] provide a competitive and robust algorithm to solve the system reliability problem. The authors use a penalty guided algorithm which searches over feasible and infeasible regions to identify a final, feasible optimal, or near optimal, solution. The penalty function is adaptive and responds to the search history. The GA performs very well on two types of problems: redundancy allocation as originally proposed by Fyffe, et al., and randomly generated problems with more complex configurations. For a fixed design configuration and known incremental decreases in component failure rates and their associated costs, Painton and Campbell [19] also used a GA based algorithm to find a maximum reliability solution to satisfy specific cost constraints. They formulate a flexible algorithm to optimize the 5th percentile of the mean time-between-failure distribution. In this paper ant colony optimization will be modified and adapted, which will consider the measure of criticality will gives a guidance to the ants for its nest and ranking of critical components will be taken into consideration to choose the most reliable components which then will be improved till reach the optimal system's components reliability value

2.2 Ant colony optimization approach

Ant colony optimization (ACO) algorithm [20, 21], which imitate foraging behavior of real life ants, is a cooperative population-based search algorithm. While traveling, Ants deposit an amount of pheromone (a chemical substance). When other ants find pheromone trails, they decide to follow the trail with more pheromone, and while following a specific trail, their own pheromone reinforces the followed trail. Therefore, the continuous deposit of pheromone on a trail shall maximize the probability of selecting that trail by next ants. Moreover, ants shall use short paths to food source shall return to nest sooner and therefore, quickly mark their paths twice, before other ants return. As more ants complete shorter paths, pheromone accumulates faster on shorter paths and longer paths are less reinforced. Pheromone evaporation is a process of decreasing the intensities of pheromone trails over time. This process is used to avoid locally convergence (old pheromone strong influence is avoided to prevent premature solution stagnation), to explore more search space and to decrease the probability of using longer paths. Because ACO has been proposed to solve many optimization problems [22],[23], our proposed idea is also to adapt this algorithm to optimize system reliability and specially complex system

3. Methodology

3.1 Problem definition

3.1 .1 Notation

In this section, we define all parameters used in our model.

Rs: Reliability of system

Pi: Reliability of components i.

qi: probability of failure of components (i).

Qn: Probability of failure to system

n : Total number of components.

ICRi: Index of criticality measure.

ICRp : index of criticality for path to destination

ISTi : Index of structure measure.

Ct : Total cost of components.

Ci: Cost of component.(production, design and maintenance costs).

Cc :cost for improvement

P(i)min: Minimum accepted reliability components

ACO

i :start node for ant,

j : next node chosen.

τ_i :initial pheromone trail intensity

$\tau_{ij}(old)$:pheromone trail intensity of combination before update of

$\tau_{ij}(New)$:pheromone trail intensity of combination after update

P_{ij} :problem-specific heuristic of combination

η_{ij} :relative importance of the pheromone trail intensity

α :relative importance of the problem-specific heuristic for global solution

β : index for component choices from set AC trail persistence for local solution

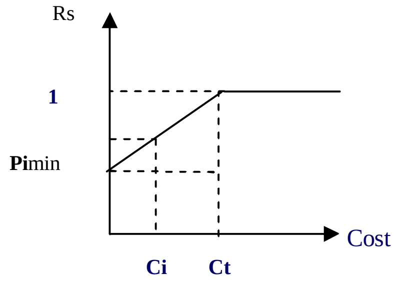
ρ : number of best solutions chosen for offline pheromone update index

3.1.2 Assumption

In this section, we present the assumptions under which formulation of our model is presented.

1: There are many different methods used to derive the expression of total reliability of complex system, which are derived in a certain system topology, we state our system expressions according to the methods of papers [3-5].

2: We used a cost-reliability curve [7] to derive an equation to express each cost components according to its reliability and then the total system cost will be additive in term of cost at constitute components. see figure(1).



Fig(1):cost-reliability curve

As show in figure (1) and by equaling the slopes of two triangles we can derive the following equation

$$Cc \geq \frac{p_1 - p(i)min}{1 - p(i)min} \cdot Ct + \frac{p_1 - p(i)min}{1 - p(i)min} \cdot Ct + \frac{p_1 - p(i)min}{1 - p(i)min} \cdot Ct + \ldots n.$$

3: We calculate the ICRi for each components from its structural measure, which given by,

ICRi =(ISTi * qi /Qn)

Where; ISTi = (∂ Rn / ∂pi) . [3] .

4-Every ICRi must be lower than initial value ai. This value is a minimum accepted level of criticality measure to every component.

5-After the complex system presented mathematically, a set of paths will be available from specified source to destination. those paths will be ranked each one according to its components criticalities.

3.2 Formulation of the problem:

The objective function in general, has the form :

Maximize,

Rn= f(P1,P2,P3,....Pn).

subject to the following constrains,

1. ICRi : i =1,2,…n
2. To ensure that the total cost of components not more than proposed cost value the following equation can be used:

$$\sum_{n=1}^{n}\left(\frac{Pi-Pimin}{1-Pimin}\right) < Ct$$

Note that this set of constrains permits only positive components cost.

4. Model Construction:

The algorithm uses an ACO technique with the criticality approach to ensure global converges from any starting point. The algorithm is iterative. At each iteration, the set of ants are identified using some indicator matrices. Below are the main steps of our proposed model . As we see in the figure (2) which illustrating a set of steps illustrated below:

1. Ant colony parameters are initialized
2. The criticality of components will be calculated according to derived reliability equation, then will be ranked according to it's values
3. Using Ant equation:

$$P_{ij} = \frac{[\tau_{ij}(t)]^{\alpha}\,[\eta_{ij}]^{\beta}}{\sum_{i\in Ni}[\tau_{ij}(t)]^{\alpha}\,[\eta_{ij}]^{\beta}}$$
$$:\eta_{ij} = \frac{1}{ICRi}$$

The probability to choose the next node will be estimated after a random number generated. and until the destination node. the selected nodes will be chosen .According to the criticality components through this path.

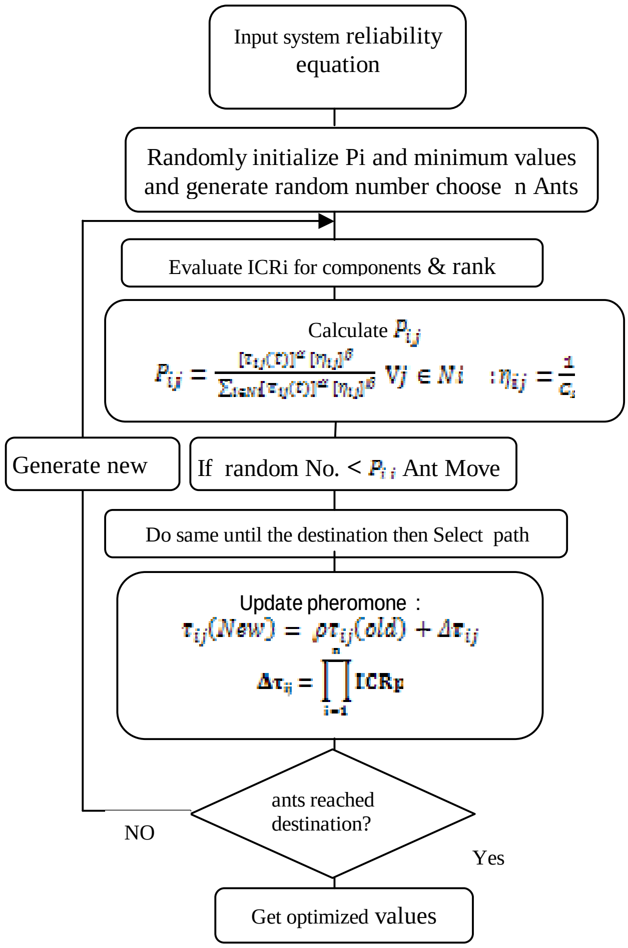


Fig2: Flow diagram of AS

4. Update the pheromone according to the criticality measure. Which can be calculate product of components criticalities' values

$$\Delta\tau_{ij} = \prod_{i=1}^{n} ICRp$$

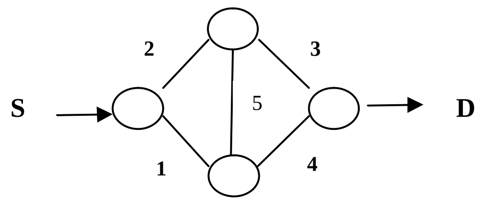
The update equation will become as follows:
$\tau_{ij}(New) = \rho\tau_{ij}(old) + \Delta\tau_{ij}$

5. A new components' reliabilities will be generated.
6. Till reach best solution and all ant moved to achieve maximum reliability of the system with minimum cost.

5. Experimental Results:

In the following examples, we use a bench mark systems configurations like a Bridge, and Delta .

5.1 Case study I:



Fig(3): Bridge system example.

To find the polynomial for a complex system we must know that it always given at a certain to be transmitted from source (s) to destination (D), see figure(3)

The objective function to be maximized has the form:

Rs=1-(q1+q4.q5.p1+q3.q4.p1.p5+ q2.q4.p1.p5.p3).

1. The ICRi constrain.

ICRi calculated : i=1,2,…5..

2- We use the values in the figure (3) as initial values for components' reliabilities to improve the system:

p(1)min=0.9, p(2)min=0.9, p(3)min=0.8, p(4)min=0.7 and p(5)min=0.8.

3. We choose the cost-reliability curve to permit distribution of cost depending on ranking of components according to there criticality.

The model was built in such a way that reduce the fail of the most critical components, this is done by increasing the reliability of the most critical components, which tend to maximizes the over all reliability what is our goal.

We summarized our results in the following table (1) and table (2).With initial values of ant colony algorithm as in table (3).

Table(1): Components and system reliabilities

Reliab-ities	New values	ICRi rank
p1	0.9998	1
p2	0.9	3
p3	0.8	4
p4	0.9998	2
p5	0.8	5
Rs	0.9999	

Table(2): The Components and system cost

cost	Value in units
C_1	9.9988
C_2	8.8888
C_3	7.7777
C_4	9.9978
C_5	7.7777
C_t	44.441

Table(3): ACO initial values

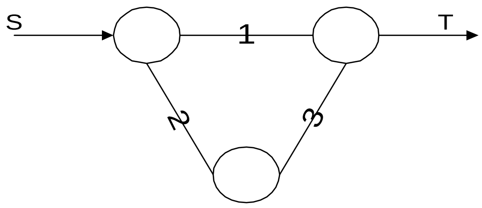
α	2
β	3
ρ	0.2
τ_{ij}	1
Q	10
Ants	10

5.1.1Comments on results

As cleared in tables 2 and 3 results indicate that according the criticality of components, the improvement will be occurred as the more critical component the more chance to be improved which will highly effect to the system reliability improvement with minimal cost too, this is better than to increase reliability components randomly. Now it is clear also the best path from S to D is to follow component 1 and component 4 . if we have more available cost it will increase the other component reliability according to it's criticality ranking.

Finally if all components have the same initial reliability values the path through components1and4 have the same chance for path through component 2 1nd 3, and according algorithm which depend on the topological reliability it will goes to improve the higher critical component according to it's position in the system.

5.2 Case study II:



Figure(4)Delta system

Using the same procedures as in case(1) we obtain the following optimization problem for delta system given in figure (4).

Max .Rn= P1+ P1.P2 - P1.P2.P3 .

Subject to

1. ICRi calculated for **i=1,2,3.**

$$\sum_{i=1}^{3} Ci*(pi) \leq 5.4$$

p(1)min=0.7 **i=1,2,3.**

The following two table (4) and (5) summarized the results.

Table(4): Components and system probabilities

	Computed value	ICRi Rank
p_1	0.9999	1
p_2	0.7	2
p_3	0.7	3
Rs	0.9999	

Table (5): Components and systems costs

	Cost values
C_1	0.9998
C_2	0.4
C_3	0.4
C_t	1.799

Beside comments noted in bridge system, delta system have two paths from S to T as shown in the figure(4). The results shows that it is preferred to increase the component one rather than others this for two reasons, it have most critical value and pheromone value biased toward the path with lower number of components according to the equation

$$\Delta\tau_{ij} = \prod_{i=1}^{n} ICR_p$$

5.3 Important Comments

To study the effect of modifying of ant parameters such as initial pheromone in a delta case and biased to component 2 the results will become as shown in table 6. The reliably for components was p1=0.2, p2=0.3 and p3=0.3 and values of α =10 , β=2 and τ_2=10

Table (6): Ant colony parameters effects

	Cost values	
C_1	0.7777	
C_2	0.9997	
C_3	0.999	
C_t	14.777	
	Computed value	**ICRi Rank**
p_1	0.3	1
p_2	0.9999	2
p_3	0..9999	3
Rs	0.9999	

It is clear that the solution biased to the components 2 and 3 path rather than component one, because of there initial pheromone values.

6. Conclusion

We propose a new effective algorithm for general reliability optimization problem. Using ant colony. The ant colony algorithm is a promising heuristic method for solving complex combinatorial problems.

To solve complex system design problem:

1. We must formulate a system, that is correctly representing the real system with all paths from source to destination

by choose an efficient reliability estimation method.
2. To the best of maximization of total reliability and minimization of the total cost of a system take in to consideration the components according to its criticality, then arrange the most critical components gradually.
3. Index of criticality achieve maximum system reliability with minimum cost according to reliability of system topology
4. resolve model without index of criticality maximum reliability and minimum cost but this method ignore the topology of the system.
5. The ant colony algorithm improved by the previous experience which was given by the index of criticality which gives to ant an experience to deposit of pheromone on a trail which will maximize the probability of selecting that trail by next ants. Moreover, ants shall use more reliable paths. Our numerical experiences show that our approach is promising especially for complex systems.

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