Modified Cubic Threshold Denoising Technique using Curvelet Transform

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ABSTRACT

Image denoising is an essential requirement of image processing. The images contain strongly oriented harmonics and edge discontinuities. Wavelets, which are localized and multiscaled, do better denoising in single dimension using multiple local thresholding technique. But because of their poor orientation selectivity they do not represent higher dimensional singularities effectively. Curvelet based denoising and reconstruction exhibit higher quality recovery of edges and curvilinear features than wavelet based constructions. In this work a modified thresholding scheme using multiple thresholds for digital curvelet transform coefficients is proposed. This thresholding scheme denoises images embedded in white Gaussian noise. The experiment shows denoising using modified cubic thresholding and curvelet transform outperforms wavelet-based methods not only in terms of PSNR(peak signal-to-noise ratio) and MSE (mean square error), but also in better visual appearance of the resulting images.

Key words: wavelet Transform, Curvelet Transform, Denoising, Soft Thresholding

1. Introduction

In the real world signals do not exist without noise, which arises during image acquisition (digitization) and/or transmission [2]. When images are acquired using a camera, light levels and sensor temperature are major factors which affect the amount of noise. During transmission images are corrupted mainly due to interference in the transmission channel [6]. The noise removal takes place in the original time space domain or in a transform domain. In transform domain, Fourier transform are used in the time–frequency domain and multiresolution transforms like wavelet/ curvelet/ contourlet transform are used in the time-scale domain.

Denoising a given noise corrupted signal is a traditional problem in both statistical and in signal processing applications. Linear denoising methods are not so effective when transient non-stationary wideband components are involved, since their spectrum is similar to the spectrum of noise [1]. Non-linear denoising methods [9] rely on the basic idea that the energy of a signal will often be concentrated in a few coefficients in the transform domain while the energy of noise is spread among all coefficients in transform domain. Therefore, the non-linear methods will tend to keep a few larger coefficients representing the signal while the noise coefficients will tend to reduce to zero. Denoising methods based on multiresolution transforms involves three steps: a linear forward transform, nonlinear thresholding step and a linear inverse transform. Wavelets are successful in representing point discontinuities in one dimension, but less successful in two

dimensions. As, multiscale representation is suited for edges and other singularity curves, the curvelet transform has emerged as a powerful tool. The developing theory of curvelets predicts that, in recovering images which are smooth away from edges, curvelets obtain smaller asymptotic mean square error of reconstruction than wavelet methods [3].
The remainder of the paper is organized as follows: A review of curvelet theory is presented in Section 2. The proposed algorithm is presented in Section 3. Experimental results and discussions are given in section 4. Finally, Section 5 summarizes the conclusions drawn from previous sections with suggestions for future work.

2. Curvelet Transform

A special member of the emerging family of multiscale geometric transforms is the curvelet transform which overcomes inherent limitations of traditional multiscale representations such as wavelets. Conceptually, the curvelet transform is a multiscale pyramid with many directions and positions at each length scale, and needle-shaped elements at fine scales [4]. Curvelets have useful geometric features that set them apart from wavelets. Curvelets obey a parabolic scaling relation which says that at scale 2^{-j}, each element has an envelope which is aligned along a "ridge" of length $2^{-j/2}$ and width 2^{-j}. Curvelet transform uses the ridgelet transform as a component step and implements curvelet subbands using a filter bank of wavelet filters. This transform combines multiscale ridgelets with a spatial bandpass filtering operation to isolate different scales [4]. While ridgelets have global length and variable widths, curvelets in addition to a variable width have a variable length and so a variable anisotropy. Curvelets occur at all scales, locations, and orientations and hence they can be used to represent a curve as a superposition of functions of various lengths and widths obeying the scaling law: width ≈ length^2.
The flow graph in Figure (1) gives an overview for organization of the algorithm. The decomposition is the sequence of following steps.
1.Subband decomposition: The image f is decomposed into subbands such that $f \to (P_0 f, \Delta_1 f, \Delta_2 f, \ldots)$.
2.Smooth partitioning: Each subband is then smoothly windowed into squares.
3.Renormalization: Each resulting square is renormalized to unit scale
4.Ridgelet analysis: Each square is analyzed via the discrete ridgelet transform.

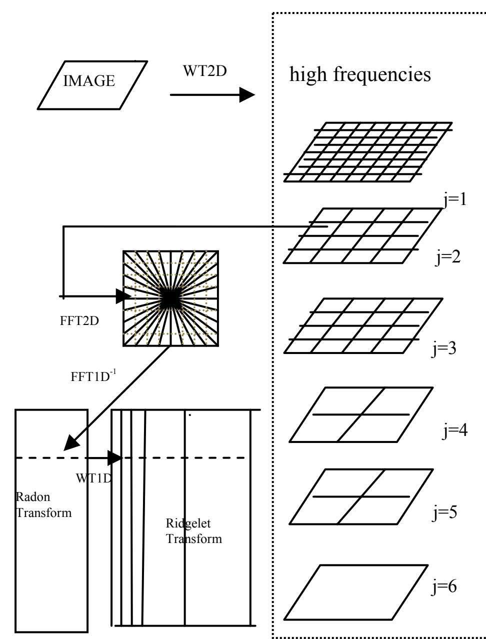

Figure1. Curvelet transform flowgraph

3. Algorithm for Image Denoising

3.1 Preliminary Algorithm for Image Denoising

The mathematical model of noisy image is as follows

$$\bar{y} = \bar{x} + \bar{n} \quad (1)$$

where $\bar{y}$ is the observed image, $\bar{x}$ the unknown original image and $\bar{n}$ the contaminating noise.

Complete curvelet denoising procedure is performed by taking curvelet transform of the image and then applying thresholding to eliminate noisy coefficients. Thus, inverse curvelet transform of thresholded coefficients give denoised image.
The fast discrete curvelet transform of the image observations are evaluated as Y using C(.) in following Equation [5].

$$Y = C(y) \tag{2}$$

The threshold denoted by λ, for wavelet is selected based on minimax threshold expressed as following

$$\lambda = D(Y) \tag{3}$$

The universal threshold [7] is given as

$$\lambda = D_g\left(Y\right) = \sigma\sqrt{2\log N} \tag{4}$$

where σ is the standard deviation of noise, and N is the size of image.
Wavelet transforms maps white noise in the signal domain to white noise in the transform domain. Thus in the transform domain the signal is concentrated into fewer coefficients but the noise does not concentrate. The principle behind separation of signal and noise is that, when scale 2^{-j} decreases, wavelet transform maxima of images doesn't increase, but at the same time wavelet transform modulus of white noise increases. Thus different behaviours of wavelet transform maxima of images and noise across different scales allow us to design the operator D(.) adaptively.

An image features a wide variety of characteristics. Hence, instead of using a single value as the global threshold, the operator D(.) can be designed to produce multiple local thresholds λ_j adaptively for different scales from fine to coarse. For wavelets one such threshold is [7]

$$\lambda_j = D_j\left(Y\right) = \sigma\sqrt{2\log N}\,/\log\left(j+1\right) \tag{5}$$

where *j* is the decomposition level of wavelet packet transform. This modified multiple local thresholding technique obtaines better results than the soft and the hard thresholding methods, which utilizes a single threshold value. Curvelet transform employs the 1-D wavelet transform as a component step, but, along the radial variable in Radon space. Thus Equation (5) does not prove to be effective for thresholding the curvelet transform coefficients and requires some modification.

3.2 Modified Cubic Thresholding for Image Denoising

In this work a similar multiple threshold technique for thresholding the curvelet coefficients is proposed. To design the operator D(.), it is proposed to retain all the coefficients at the first scale, since they are the dc values and they provide the average information of the image. For the remaining scales the coefficients, which provide the highest PSNR values seemed to be correlated and the curvelet coefficients appeared to decay in an exponential manner. Thus, a scale dependent exponential function multiplied by a scale dependent logarithmic function resulted in improvement in PSNR values. Therefore, the multiple local thresholds is proposed as follows

$$\lambda_j' = D_j\left(Y\right) = 2\sigma\sqrt{\log N}.e^{-(j-1)}.\log(j+1) \tag{6}$$

where *j* is the decomposition level of curvelet transform and varies from *j = 2,3,........J*. *J* is the integer corresponding to the last scale.
The thresholding operator T(.,.) is defined as

$$Z = T(Y, \lambda) \tag{7}$$

It is taken as the cubic threshold function [7], given as

$$T_n(Y,\lambda_j) = \begin{cases} 0, & \lambda_j \\ Y\left(1-\left|\dfrac{\lambda_j}{Y}\right|^3\right), & else \end{cases} \tag{8}$$

Cubic thresholding function is very flexible and has the capability to adapt to different types of images and threshold operators.
Some simple but powerful shrinkage functions are the soft [8], and the hard thresholds. They select a single global

threshold for all the scales using Equation (4).This work proposes the use of multiple local thresholds, λ_j'of Equation (6) with the soft and hard thresholds. Thus multiple local soft thresholding may be given by

$$T_s(Y,\lambda_j') = \begin{cases} 0, & |Y| < \lambda_j' \\ Y - \lambda_j' sign(Y), & else \end{cases} \quad (9)$$

Similarly multiple local hard thresholding is given by

$$T_h(Y,\lambda_j') = \begin{cases} 0, & |Y| < \lambda_j' \\ Y, & else \end{cases} \quad (10)$$

Finally $C^{-1}(.)$ takes the fast discrete inverse curvelet transform of the thresholded curvelet coefficients as follows

$$\hat{x} = C^{-1}(Z) \quad (11)$$

Where $\hat{x}$ is reconstructed/denoised image

4. Experiments and Results

The performance of the proposed thresholding method is evaluated and compared with soft, hard and cubic thresholding schemes using wavelets [7]. The Gaussian noise with standard deviation 26.25 is added to the classical *Lenna* image of (512x512) to obtain noisey image. The Multiple local thresholds are obtained using Equation (6). The curvelet coefficients are processed by thresholding functions in Equations (8), (9), and (10). The performance of denoising is evaluated using PSNR and MSE. PSNR is defined as the ratio of signal power to noise power. It basically obtains the gray value difference between resulting image and original image. PSNR is given as

$$PSNR = 10\log_{10}\left(\frac{MAX_I^{\,2}}{MSE}\right) \quad (12)$$

where MAX_I is the maximum pixel value of the image. MSE is given by the formula

$$MSE = \frac{1}{mn}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}\|x(i,j) - \hat{x}(i,j)\|^2 \quad (13)$$

where x is the original image, $\hat{x}$ is the reconstructed image, m and n are the number of rows and columns respectively.Numerical values for PSNR and MSE are given in Table1.

Method	Thresholding using wavelets		Multiple local thresholding using curvelets	
	PSNR / dB	MSE	PSNR / dB	MSE
Noisy Image	19.80	679.91	19.80	679.91
Soft_thresholding	23.11	317.61	25.31	191.461
Hard_thresholding	24.52	229.86	26.13	158.51
Multiple local Cubic_thresholding	25.07	202.19	26.56	143.57

Table 1.Comparison of different thresholding methods

The curvelet reconstruction using multiple local thresholds enjoys superior performance over the wavelet based reconstructions. The pictorial denoising performance using wavelet based soft, hard, and cubic thresholds is compared with curvelet based multiple local soft, hard, and cubic thresholds in Figure 2. Experiments show that multiple local thresholding based on curvelets outperforms the wavelet based methods on the basis of MSE and PSNR.

5. Conclusion

In this work a multiple local thresholding technique, using curvelets, is proposed to denoise images contaminated by white Gaussian noise. Curvelets carry multiresolution properties. At different scales the maxima of curvelet transform coefficients vary, therefore, the threshold operator D(.), is so designed, which produces multiple local thresholds, adaptively, for different scales from fine to course. The proposed expression for varying thresholds at different scales is then applied to threshold the curvelet coefficients with cubic, hard and soft thresholding functions. The effectiveness of the method is tested against the wavelet based methods. Our experimental results depict that, multiple local thresholding technique using curvelets outperform the wavelet based methods. In addition, this technique outputs images that are visually clearer with well defined edges. Further work can be carried on the issue to quantitatively analyze the denoising results according to different types and magnitudes of noises. Moreover in future, higher resolution images of size 2048x2048 or 4096x4096 will become the standards for which this work needs to be tested.

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Figure 2. Denoising of Lenna (a) Original image. (b) Noisy image. (c) Soft thresholding using wavelets. (d) Multiple local soft thresholding using curvelets. (e) Hard thresholding using wavelets. (f) Multiple local hard thresholding using curvelets. (g) Multiple local cubic thresholding using wavelets. (h) Multiple local cubic thresholding using curvelets