Optimizing Inventory Decisions in a Multi-Stage Supply with Planned Back Orders

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ABSTRACT

In this paper we consider the case of a three-stage supply where a firm can supply many customers. This supply chain system involves suppliers, manufactures, and retailers. Production and inventory decisions are made the suppliers and manufactures levels. The problem is to coordinate production and inventory decisions across the supply chain so that the total cost of the system is minimized. For this purpose we formulate a model for the equal cycle time inventory coordination mechanism with planned back orders. A numerical example is presented to illustrate the model.

Key Words: Supply chains, inventory coordination, optimization, planned stock outs.

1. Introduction

Supply chains are complex systems that involve suppliers, manufactures, and retailers. The main objective of supply chain management (SCM) is to efficiently integrate suppliers, manufactures, warehouses, and stores, so that merchandise is produced and distributed at the right quantities, to the right locations, and at the right time, in order to minimize system-wide cost while satisfying service level requirements.

Recently numerous articles in supply chain modeling have been written in response to the global competition. Banerjee (1986) introduced the concept of joint economic lot sizing problem (JELS). He considered the case of a single vendor and a single purchaser under the assumption of deterministic demand and lot for lot policy. He analyzed the effects of each party's optimal lot size on the other in case of independent optimization and developed a JELS model that focused on the joint total relevant cost (JTRC). Goyal & Szendrovits (1986) presented a constant lot size model where the lot is produced through a fixed sequence of manufacturing stages, with a single setup and without interruption at each stage. Transportation of partial lots, called batches, is allowed between stages. This model mainly, relaxes the constraint that batches must be of equal size at any particular stage. Goyal (1988) provided a more general model for the case of single vendor single buyer through relaxing the lot-for-lot policy. He assumed that the whole production lot should be produced before shipments take place. He showed that his model provides a lower or equal total joint relevant cost compared to Banerjee (1986).

Goyal and Gupta (1989) extensively reviewed the literature which deals with the interaction between a buyer and vendor. They classified the literature dealing with the integrated models into four main classes. The first class represents models which deal with joint economic lot sizing policies. The second class characterizes models which deal with the coordination of inventory by simultaneously determining the order quantity for the buyer and the vendor. The third class is a group of models which deal with integrated problem but do not determine simultaneously the order quantity of the buyer and the vendor. The last class represents models which deal with buyer vendor coordination subject to marketing considerations.

Lu (1995) developed a one-vender multi-buyer integrated inventory model with the objective of minimizing the vender's total annual cost subject to the maximum cost that the buyer may be prepared to incur. Lu has found the optimal solution for the single-vendor single-buyer case under the stated assumptions and presented a heuristic approach for the one-vendor multi-buyer case. Goyal (1995) in his short paper revisited the single-vendor single-buyer where he relaxed the constraint of equal sized shipments of Goyal (1988) and suggested that the shipment size should grow geometrically. Lee (2005) considered a single-manufacturer single-supplier supply chain where the manufacturer orders its raw materials from its supplier, the converts the raw materials into finished goods, and finally delivers the finished goods to its customers. He proposed an integrated inventory control model that comprises of integrated vendor-buyer (IVB) and integrated procurement-production (IPP) systems.

Hoque & Goyal (2000) Extended the idea of producing a single product in a multistage serial production system with equal and unequal sized batch shipments between stages. Khouja (2003) considered the case a three-stage non-serial supply chain and developed the model to deal with three inventory coordination mechanisms between the chain members. Bendaya & Nassar (2005) relaxed the assumption of Khouja (2003) regarding the completion of the whole production lot before making shipments out of it and assumed that equal sized shipments take place as soon as they are produced and there is no need to wait until a whole lot is produced. In this paper, we extend Khouja (2003) by assuming that the end retailers can backlog orders.

The remainder of this paper is organized as follows. The next section presents problem Definition, Notations and assumptions. Section 3 describes the development of the model. Section 4 presents a numerical example. Finally, section 5 contains some concluding remarks.

2. Problem Definition and Notations

Consider the case of a three-stage supply chain where a firm can supply many customers with a single product. This supply chain system involves suppliers, manufactures and retailers. Production and inventory decisions are made the suppliers and manufactures levels. The production rates for the suppliers and manufactures are assumed finite. In addition the demand for each firm is assumed to be deterministic. Unsatisfied demands at the end retailers are backordered. The problem is to coordinate production and inventory decisions across the supply chain so that the total cost of the system is minimized.

T =Basic equal cycle time,

TS = stock out duration time

Ai =Setup cost at stage i

hi =Inventory holding cost at stage i

$\hat{h}$ =Shortage cost per unit short, independent of the duration of the shortage

$\hat{p}$ = Shortage cost per unit short per unit time

n_i =Number of firms at stage i

D_{ij} =The mean demand of firm j at stage i

P_{ij} =Production rate of firm j at stage i

3. Model Development

Let i=1,2, and 3 denote the stage index in the supply chain. And let j=1,2,…, J_i be an index denoting firms within each stage. The expected total cost per unit time for a downstream retailer can be formulated as:

$$TC_{3,j} = h_3 \frac{D_{3,j}(T-T_S)^2}{2T} + p\frac{D_{3,j}T_S^2}{2T} + h\frac{D_{3,j}T_S}{T} + \frac{A_3}{T} \quad (1)$$

Expected total cost for a firm at stage 2 (i.e. manufacturer) is:

$$TC_{2,j} = \frac{TD_{2,j}^2}{2P_{2,j}}(h_1+h_2) + \frac{A_2}{T} \quad (2)$$

Similarly the expected total cost for a firm at stage 1 (i.e. supplier) is:

$$TC_{1,j} = \frac{TD_{1,j}^2}{2P_{1,j}}(h_0+h_1) + \frac{A_1}{T} \quad (3)$$

Now the expected total per unit time cost for the entire supply chain is:

$$TC = \sum_{ij} TC_{ij} \quad (4)$$

Taking the first derivative of the total annual cost of the whole supply chain and solving $\partial TC/\partial T = 0$ gives the following optimal cycle time:

$$T = \sqrt{j(T_S)/f} \quad (5)$$

Where:

$$j(T_S) = (DT_S^2 h_3 + DT_S^2 p + 2DT_S h + 2(J_1A_1 + J_2A_2 + J_3A_3)) \quad (6)$$

and

$$f = \sum_1^{J_1} \frac{D_{1,j}^2}{2P_{1,j}}(h_0+h_1) + \sum_1^{J_2} \frac{D_{2,j}^2}{2P_{2,j}}(h_1+h_2) + Dh_3 \quad (7)$$

To minimize TC, the solution to $dTC/\partial T_S = 0$ gives:

$$T_S = \frac{Th_3 - h}{h_3 + p} \quad (8)$$

Substituting for T_S from Eq. (8) back into TC, and the solving $\partial TC/\partial T = 0$ gives an An initial value of T , and this value is used to compute an initial value for T_S using Eq. (8) and this value is used to compute another value for T using Eq. (5). This computation procedure is continued iteratively until successive values of T and T_S are sufficiently close.

4. Numerical results

In this section, we consider an example of a three stage supply chain having one supplier, one manufacturer, and four retailers. The relevant data is shown in Table 1.

Retailers					
j	A_i	h_i	p	h	D
1	10	8	0.3	11	150
2	10	8	0.3	11	200
3	10	8	0.3	11	150
4	10	8	0.3	11	200
Manufacturer					
j	A_i	h_i	D	P	
1	20	6	700	1000	
Supplier					
1	30	4	700	1000	

Table 1: Example data

The results for this example are summarized in Table 2. These results were obtained using the computation procedure presented in section 3.

Stage	T	Ts	Cost
Retailers	0.16	0.05	199.78
Manufacturers	0.16	-	222.82
Supplier	0.16	-	2467.11
Entire chain	0.16	0.05	2889.71

Table 2: Results

5. Conclusion

In this paper we consider the case of a three-stage supply chain. This supply chain system involves suppliers, manufactures, and retailers. Production and inventory decisions are made the suppliers and manufactures levels. We formulated a model to deal with the same cycle time inventory coordination mechanisms between the chain members, with planned backorders at the end retailers.

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