Extended Cavity Model to Analysis Tunable Circular Disk Microstrip Antenna Using Genetic Algorithm

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Abstract—In this paper, the cavity model for simple circular disc microstrip antenna is extended with some modifications for the tunable geometry taking into account the anisotropy in the layer. The numerical results show that there are substantial deviations in calculated resonant frequency when substrate dielectric anisotropy is considered. Furthermore, significant variations are seen in the radiation patterns of the structures due to substrate anisotropy. Finally the effect of inclusion of air gap layer inserted between substrate and ground plane on the resonant characteristics is also investigated for fundamental and higher order modes.

Keywords—cavity model; genetic algorithm; anisotropic substrate; adjustable air gap.

I. Introduction

Microstrip antennas are becoming increasingly popular since they have small volume and a low-profile planar configuration. Easy mass production of such antennas using printed circuit technology leads to low fabrication cost. They are much easier to be integrated into microwave circuits on the same substrate. Especially, they can be made conformal to the host surface [1].

Some dielectric substances exhibit anisotropy due to their natural crystal structures or as the result of their production processes. Isotropic substances may also exhibit anisotropy at high frequencies. In the design of microwave integrated circuit components and microstrip antennas, anisotropic substances have been increasingly popular [2-10]. Uniaxial substrates have drawn more attention due to their availability such as sapphire, boron nitride and E-10 ceramic-impregnated Teflon. Their main drawback is narrow bandwidth characteristics, which is considerably avoided by operating the antenna around the resonant frequency. As an alternative, double-layered structure with air gap having adjustable thickness between the substrate and the ground plane is also found to be useful in obtaining the wide band operation. For both single and double-layered structures, accurate computation of resonant frequency is an important task and takes considerable interest in literature by various authors depending on the usage of various methods and approximations [11–19]. In this study, resonant frequency of double layered circular patch microstrip antenna is accurately determined via cavity analysis, using a simple effective permittivity and patch radius expressions including modal effects. The aim of this work to perform an accurate and efficient analysis of circular-disc microstrip antennas on double layer, as well as to perform the analyses for circular microstrip antennas on a single layer substrate and on a tunable substrate, as particular cases.

II. Antenna Configuration and Design

The tunable circular microstrip antenna structure is shown in Fig. 1. The resonant behavior of the antenna is independent of the feed so that the feed was not taken into account in the analysis.

![Fig. 1. Geometry of a circular microstrip antenna with air gap.](image-url)
The resonant frequency of this antenna can be determined from cavity model for various operational modes and structural parameters using proper equivalent model with effective structural parameters [3]. For this purpose, various expressions for effective patch radius $a_{eff}$ and effective relative permittivity $\varepsilon_{eff}$ are defined in literature [13, 15, 18, 19]. In this study, effective patch radius expression to approximate the modal effects is taken for the double-layered antenna in the modified form:

$$ f_{r} = \frac{\chi_{mn} v_{0}}{2 \pi a \sqrt{\varepsilon_{req}}} \quad (1) $$

where $\chi_{mn}$ is the mth zero of the derivative of the Bessel function of order n, the value of which $(\chi_{21}=3.832, \chi_{11}=1.841, \chi_{31}=3.054, \chi_{13}=4.201)$ determines the lowest and higher order modes as $TM_{11}$, $TM_{21}$, $TM_{01}$, and $TM_{31}$ modes. $v_{0}$ is the velocity of light in free space, $a$ is the patch radius, and $\varepsilon_{req}$ is the substrate relative permittivity of the equivalent structure which can be determined from the cavity model [14]

$$ \varepsilon_{req} = \varepsilon_{r2}(d_{1} + d_{2})/(\varepsilon_{r2}d_{1} + d_{2}) \quad (2) $$

To account for the fact that small fraction of the field exists outside the dielectric; it is customary to use effective permittivity $\varepsilon_{eff}$ in place of $\varepsilon_{req}$

$$ \varepsilon_{eff} = \varepsilon_{req} - 0.9\varepsilon_{req} \left[ \frac{2d_{2}}{a} + \left( \frac{d_{2}}{a} \right)^{2} \right] \quad (3) $$

Where $d = d_{1} + d_{2}$ and, $\varepsilon_{r2}$ is the relative permittivity of dielectric substrate.

If we want to take the substrate uniaxial anisotropy’s into account, the relative dielectric permittivity $\varepsilon_{r}$ will be replaced with the tensor $\varepsilon_{r} = \text{diag}(\varepsilon_{x}, \varepsilon_{x}, \varepsilon_{z})$ where $\varepsilon_{x}$ and $\varepsilon_{z}$ are the relative dielectric permittivity along $x$ and $z$ axis, respectively.

- For the case of isotropic substrate with air gap, we use the effective dielectric constant $\varepsilon_{eq}$ given in Eq. (2).
- For the case of uniaxially anisotropic substrate without air gap, $\varepsilon_{eq}$ given in [21] Eq. (2) is used to determine $d_{e}$, there resulting values are:

$$ \varepsilon_{req} = \varepsilon_{z} \quad (4) $$

$$ d_{e} = d \frac{\varepsilon_{x}}{\varepsilon_{z}} \quad (5) $$

To account fringe field effects, the circular patch radius $a$ given in Eq. (1) should be replaced by its effective value [20, 21]. In this letter, a new effective patch radius expression is presented to compute the resonant frequency of a circular MSA with thin and without air gap for providing better accuracy. By utilizing the experimental data reported elsewhere [22-27], after many trials, the following model, depending on $\varepsilon_{eff}$, $a$ and $d$, which produces good results, was chosen

$$ a_{eff} = a + \left[ \beta_{1} + \left( \frac{\beta_{2}}{\varepsilon_{eff} r_{p}} \right) \right] d + \left( \frac{\beta_{4}}{a} \right) d^{2} \quad (6) $$

where the unknown coefficients are determined by a genetic optimization algorithm. It is evident from (6) that the effective patch radius, $a_{eff}$ is larger than the physical patch radius, $a$, provided the conditions and are satisfied. In the following section, the genetic optimization algorithm used in this work is described and then the application of the genetic algorithm to the problem is explained.

### III. Genetic Algorithm

The GA [28, 29] is based on the evolution theory where weak species face extinction but strong ones survive and pass their genes to the next generation. However for the strong species to survive there is also a requirement for random injection of genes. As GA mainly manipulates matrices it is normally implemented using Matlab software. The step by step procedure of generating the software program is shown below.

**Step 1:** Each variable is assigned a number of binary digits so that the required accuracy of this variable is obtained in the final solution.

**Step 2:** All the variables in their binary form are grouped into a string which is called a chromosome.

**Step 3:** Matlab is used to select a fixed number of random chromosomes called a population out of all possible number of chromosomes that are present. This is called the current generation.

**Step 4:** Converting the digital value of each variable in a chromosome to an analogue value, the objective function (F) is evaluated and the relative fitness of each chromosome ($P_{i}$) determined. This relative fitness is defined as:

$$ F = \sum_{i=1}^{n} \text{eval}_{i}[P_{i}] \quad (7) $$

**Step 5:** The selective probability is determined by:
The cumulative probability of the chromosomes is given as:

$$q_i = \sum_{j=1}^{n} p_{sj}$$  \hspace{1cm} (9)$$

Then a random number 'r' is generated in the range 0 to 1. If $q_{i-1} \leq r \leq q_i$ then select $P_{si}$.

**TABLE I.** RESULTS AND COMPARISON OF THE RESONANT FREQUENCIES OF MEASURED AND CALCULATED FOR THE FUNDAMENTAL MODE TM11 OF A CIRCULAR ANTENNA AND THE NO GAP CASE.

<table>
<thead>
<tr>
<th>Physical and Electrical Parameters</th>
<th>Measured $f_r$ (GHz)</th>
<th>Calculated Frequencies $f_r$ (GHz)</th>
<th>Our results (GHz) $f_r$</th>
<th>Measured By</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$ (mm)</td>
<td>$\varepsilon_r$</td>
<td>$a$ (mm)</td>
<td>[23]</td>
<td>[26]</td>
</tr>
<tr>
<td>1.5875</td>
<td>2.5</td>
<td>34.93</td>
<td>1.57</td>
<td>1.592</td>
</tr>
<tr>
<td>3.175</td>
<td>2.5</td>
<td>34.93</td>
<td>1.51</td>
<td>1.592</td>
</tr>
<tr>
<td>2.35</td>
<td>4.55</td>
<td>49.5</td>
<td>0.825</td>
<td>0.832</td>
</tr>
<tr>
<td>2.35</td>
<td>4.55</td>
<td>29.9</td>
<td>1.36</td>
<td>1.378</td>
</tr>
<tr>
<td>2.35</td>
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<td>3.75</td>
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</tr>
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<td>4.425</td>
<td>4.695</td>
</tr>
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<td>2.65</td>
<td>8.2</td>
<td>6.074</td>
<td>7.297</td>
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</table>

**TABLE II.** COMPARISON OF THE RESONANT FREQUENCIES OF MEASURED AND CALCULATED OF A CIRCULAR ANTENNA HAVING AN AIR GAP; $a = 50$mm, $\varepsilon_r = 2.32, d_2 = 1.59$mm.

<table>
<thead>
<tr>
<th>Mode TM, $d_1$ (mm)</th>
<th>Measured $f_r$ (GHz)</th>
<th>Calculated Frequencies $f_r$ (GHz)</th>
<th>Our results (GHz) $f_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5</td>
<td>1.286</td>
<td>1.298</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1.350</td>
<td>1.368</td>
</tr>
<tr>
<td>TM$_{21}$</td>
<td>0</td>
<td>1.879</td>
<td>1.927</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>2.136</td>
<td>2.167</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2.256</td>
<td>2.280</td>
</tr>
<tr>
<td>TM$_{31}$</td>
<td>0</td>
<td>2.596</td>
<td>2.665</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>2.951</td>
<td>2.994</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3.106</td>
<td>3.150</td>
</tr>
</tbody>
</table>
The unknown coefficient values of the model given by (6) are optimized by the genetic optimization algorithm just described. The optimum values found are

\[ \beta_1 = 0.12, \quad \beta_2 = 2.54, \quad \beta_3 = 3.65, \quad \beta_4 = 0.23 \]  \hspace{1cm} (11)

The effective patch radius expression, \( a_{\text{eff}} \), is obtained by substituting the coefficient values given by (11) into (6).

**IV. Results and Discussion**

In order to determine the most appropriate suggestion given in the literature, we compared our computed values of the resonant frequencies for the fundamental mode of circular microstrip antenna with the theoretical and experimental results reported by other scientists [22, 23, 26, and 27], all of which are given in Table I.

In order to check the accuracy of the model for two-layered case, the results are compared with an experimental and the theoretical values presented in the previous work [12, 13, 16, 17, and 30] in Table II.

Figure 3 show the resonant frequency against the air gap thickness for several radius values of the circular-disc patch. It is seen that the operating frequency increases with the air layer thickness for a given value of patch size. So, antenna tuning is possible by introducing the air gap without changing the antenna parameters.

![Graph showing resonant frequency versus air gap thickness for different values of patch radius](image)

**Fig. 3.** Resonant frequency versus air gap thickness for different values of patch radius, \( d_i = 1.27 \text{ mm} \), and \( e_x = 2.32 \).

It is observed that when the air separation grows, the resonant frequency increases rapidly until achieving a maximum operating frequency at a definite air separation \( d_{i_{\text{max}}} \). Note that the effect of the air gap is more pronounced for small values of \( d_i \) show “Figure. 3”. When the air separation exceeds \( d_{i_{\text{max}}} \), increasing the air gap width will decrease slowly the resonant frequency. These behaviors agree with those discovered theoretically for resonant frequency of circular patch antenna [16-19]. however, it depends inversely on the patch size for a given air gap width \( d_i \).

Next, the effect of uniaxial anisotropy on the resonant frequency is analyzed.

Fig. 4 depicts the influence of the patch radius on the resonant frequency of a circular microstrip antenna for anisotropic dielectric substrates (without air gap): Boron nitride \( (e_x = 5.12, e_z = 3.4) \). The substrate has thickness \( d = 1.27 \text{ mm} \). As it can be seen, the resonant frequencies reduce considerably with the dielectric substrates of Boron nitride.

![Graph showing resonant frequency as a function of radius patch of a circular microstrip antenna without air gap on anisotropic substrate](image)

**Fig. 4.** Resonance frequency as a function of radius patch of a circular microstrip antenna without air gap on anisotropic substrate, \( (e_x = 5.12, e_z = 3.4) \), \( d = 1.27 \text{ mm} \).

Also it observed that the resonant frequency increases with the patch radius.

Fig. 5 depict the influence of the air gap thickness on the resonant frequency of a circular-disc microstrip patch for three anisotropic dielectric substrates: Boron nitride \( (e_z = 3.4, e_x = 5.12) \), Epsilam-10 \( (e_z = 10.3, e_x = 13) \), and Sapphire \( (e_z = 11.6, e_x = 9.4) \). The substrate has thickness \( d_i = 1.27 \text{ mm} \) and the air gap width is varied from 0 mm to 5 mm.

As it can be seen, the resonant frequency reduces considerably when the dielectric substrate changes from Boron nitride to Epsilam-10, and this is in contrast to what happens when the medium changes from Epsilam-10 to Sapphire. The obtained results show that when the permittivity \( e_z \) is changed and \( e_x \) remains constant, the resonant frequency changes drastically, on the other hand, we found a slight shift in the resonant frequency.
when the permittivity $\varepsilon_x$ is changed and $\varepsilon_z$ remains constant. These behaviors agree very well with those reported by [6]. Also it is observed that the resonant frequency increases with the air gap thickness.

![Graph showing resonant frequency versus air gap thickness for different anisotropic dielectric substrates.](image)

Fig. 5. Resonant frequency versus air gap thickness for different anisotropic dielectric substrates.

**V. Conclusion**

In this paper, a simple CAD formulation is presented based the cavity model for simple circular disc microstrip antenna is extended with some modifications for the tunable geometry taking into account the anisotropy in the layer. Computations show that the air separation can be adjusted to have the maximum operating frequency of the antenna. Extreme care should be taken when designing a microstrip antenna with thin air gap; since small uncertainty in adjusting the air separation can result in an important detuning of the frequency. The effects of a uniaxial substrate on the resonant frequency of structures are considered in detail. The results of the study will also be useful in the microstrip disk antenna design using uniaxial metamaterials.

**References**


