Modeling Equipment Deterioration for Dependability Analysis

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Abstract
Selecting maintenance policy is important for efficient operation of contemporary complex computer systems, when not only reliability, but also financial factors must be taken into consideration. This work presents an approach which is based on the concept of a life curve and discounted cost used to study the effect of equipment aging under different maintenance policies. The deterioration process is first described by a Markov model and then its various characteristics are used to develop the equipment life curve and quantify other reliability parameters. Based on these data, effects of various “what-if” maintenance scenarios can be estimated and their efficiency compared. Simple life curves are combined to model equipment deterioration undergoing diverse maintenance actions, while computing other parameters of the model allows evaluating additional factors, such as probability of equipment failure.

Key Words: Markov model, equipment deterioration, model adjustment

1. Introduction
Effective and efficient maintenance is a significant factor in operation of today’s complex computer systems. Selecting the optimal maintenance strategy must take numerous issues into account and among them reliability and economic factors are often of equal importance. On one side, it is obvious that for successful system operation failures must be avoided and this opts for extensive and frequent maintenance activities. On the other, superfluous maintenance may result in very large and unnecessary cost. Finding a reasonable balance between these two is a key point in efficient system operation.

This paper describes Asset Risk Manager – a computer software package for a person deciding about maintenance activities which would help to evaluate risks and costs associated with choosing different maintenance strategies. Rather than searching for a solution to a problem: “what maintenance strategy would lead to the best dependability parameters of system operation”, in this approach different maintenance scenarios can be examined in “what-if” studies and their reliability and economic effects can be estimated.

The text is organized as follows. The next section describes methodology for modeling the equipment aging that is based on a particular form of Markov models taking into account specific maintenance activities like inspections and repairs. The models form a foundation for all ARM analysis. Section 3 deals with one specific problem of model construction, namely with assuring compliance with repair frequencies that are known from real-life operational records. Finally, section 4 presents the ARM software: its operation from the user point of view, types of dependability analysis that are performed and methods for visualization of the results.

The ARM system has been initially presented in [8]. This paper extends that presentation with additional discussion of the method for Markov model adjustment and its impact on new results that can be included in the studies ([9]).

2. Modeling the aging process in the presence of maintenance activities
In the proposed approach it is assumed that the equipment will deteriorate in time and, if
not maintained, will eventually fail. If the deterioration process is discovered, preventive maintenance is performed which can often restore the condition of the equipment. Such a maintenance activity will return the system to a specific state of deterioration, whereas repair after failure will restore to “as new” condition [4-5].

2.1 Life curves
A convenient way to represent the deterioration process is by the life curve of the equipment [5]. Such a curve shows the relationship between asset condition, expressed in either engineering or financial terms, and time. Since there are many uncertainties related to the prediction of equipment life, probabilistic analysis must be applied to construct and evaluate life curves. Figure 1 (a) shows an example of a simple life curve of some equipment that models its continuous deterioration up to the point of failure. Figure 1 (b) illustrates application of this curve in a case study of a specific scenario in which equipment refurbishment and equipment failure occur.

2.2 The aging process
There are three major factors that contribute to the aging behavior of equipment: physical characteristics, operating practices, and the maintenance policy. Of these three aspects the last one relates to events and actions that should be properly incorporated in the model.

The maintenance policy components that must be recognized in the model are: monitoring or inspection (how is the equipment state determined), the decision process (what determines the outcome of the decision), and finally, the maintenance actions (or possible decision outcomes).

In practical circumstances, an important requirement for the determination of the remaining life of the equipment is the establishing its current state of deterioration. Even though at the present state of development no perfect diagnostic test exists, monitoring and testing techniques may permit approximate quantitative evaluation of the state of the system. It is assumed that four deterioration states can be identified with reasonable accuracy: (a) normal state, (b) minor deterioration, (c) significant (or major) deterioration, and (d) equipment failure. Furthermore, the state identification is accomplished through the use of scheduled inspections. Decision events generally correspond to inspection events, but can be triggered by observations acquired through continuous monitoring. The decision process will be affected by what state the equipment is in, and also by external factors such as economics, current load level of the equipment, its anticipated load level and so on.

2.3 The model
All of the above assumptions about the aging process and maintenance activities can be incorporated in an appropriate state-space (Markov) model. It consists of the states the equipment can assume in the process, and the possible transitions between them. In a Markov model the rates associated with the transitions are assumed to be constant in time.
The development described in this paper uses model of Asset Maintenance Planner [6-7]. The AMP model is designed for equipment exposed to deterioration but undergoing maintenance at prescribed times. It computes the probabilities, frequencies and mean durations of the states of such equipment. The basic ideas in the AMP model are the probabilistic representation of the deterioration process through discrete stages, and the provision of a link between deterioration and maintenance.

For structure of a typical AMP model see Figure 2. In most situations, it is sufficient to represent deterioration by three stages: an initial (D1), a minor (D2), and a major (D3) stage. This last is followed, in due time, by equipment failure (F) which requires extensive repair or replacement.

In order to slow deterioration and thereby extend equipment lifetime, the operator will carry out maintenance according to some predefined policy. In the model of Figure 2, regular inspections (I) are performed which result in decisions to continue with minor (Ms1) or major (Ms2) maintenance or do nothing. The expected result of all maintenance activities is a single-step improvement in the deterioration chain; however, allowances are made for cases where no improvement is achieved or even where some damage is done through human error in carrying out the maintenance resulting in the next stage of deterioration.

The choice probabilities (at the points of decision making) and the probabilities associated with the various possible outcomes are based on user input and can be estimated e.g. from historical records or operator expertise.

Mathematically, the model in Figure 2 can be represented by a Markov process, and solved by well-known procedures. The solution will yield all the state probabilities, frequencies and mean durations. Another technique, employed for computing the so-called first passage times (FPT) between states, will provide the average times for first reaching any state from any other state. If the end-state is F, the FPT’s are the mean remaining lifetimes from any of the initiating states.

3. Adjusting model parameters

Preparing the Markov model for some specific equipment is not an easy task and requires expert intervention. The goal is to create the model representing closely real-life deterioration process known from the records that usually describe average equipment operation under regular maintenance policy with some specific frequencies of inspections and repairs. Compliance with these frequencies in behavior of the model is a very desirable feature that verifies its trustworthiness.
This section describes a method of model adjustment that aims at reaching such a compliance ([9]). It can be used also for a different task: fully automatic generation of a model for a new maintenance policy with modified frequencies of repairs.

3.1 The method

Let \( K \) represents number of deterioration states and \( R \) number of repairs in the model under consideration. Also, let \( P^{sr} = \) probability of selecting maintenance \( r \) in state \( s \) (assigned to decision after state \( I_s \)) and \( P^{r0} = \) probability of returning to state \( D \) from inspection \( I_s \) (situation when no maintenance is scheduled as a result of the inspection). Then for all states \( s = 1 \ldots K \):

\[
p^{r0} + \sum_r p^{sr} = 1 \quad (1)
\]

Let \( F_r^{s} \) represents frequency of repair \( r \) acquired through solving the model. The problem of model tuning can be formulated as follows:

Given an initial Markov model \( M_0 \), constructed as above and producing frequencies of repairs \( F_0 = [F_0^0, F_0^1, \ldots F_0^R] \), adjust probabilities \( P^{sr} \) so that some goal frequencies \( F_G \) are achieved.

The vector \( F_G \) usually represents observed historical values of the frequencies of various repairs. In the proposed solution, a sequence of tuned models \( M_0, M_1, M_2, \ldots, M_N \) is evaluated with each consecutive model approximating desired goal with a better accuracy. The procedure consists of the following steps:

1° For model \( M_i \) compute vector of repair frequencies \( F_i \).

2° Evaluate an error of \( M_i \) as a distance between vectors \( F_G \) and \( F_i \).

3° If the error is within the user-defined limit consider \( M_i \) as the final model and stop the procedure \( (N = i) \); otherwise proceed to the next step.

4° Create model \( M_{i+1} \) through tuning values of \( P^{sr} \), then correct \( P^{r0} \) according to (1).

5° Proceed to step 1° with the next iteration.

The error computed in step 2° can be expressed in may ways. As the frequencies of repairs may vary in a broad range within one vector \( F_i \), yet values of all are significant in model interpretation, the relative measures work best in practice:

\[
|F_G - F_i| = \frac{1}{R} \sum_{r=1}^{R} \left| F'_r / F'_G - 1 \right|
\]

or

\[
|F_G - F_i| = \max_r \left| F'_r / F'_G - 1 \right| \quad (2)
\]

The latter formula is more restrictive and was used in examples of this paper.

3.2 Approximation of model probabilities

Of all the steps outlined in the previous section, it is clear that adjusting probabilities \( P^{sr} \) in step 4° is the heart of the whole procedure.

In general, the probabilities represent \( K \cdot R \) free parameters and their uncontrolled modification could lead to serious deformation of the model. To avoid this, a restrictive assumption is made: if the probability of some particular maintenance must be modified, it is modified proportionally in all deterioration states, so that at all times

\[
\hat{p}_r^{i, r} : \hat{p}_0^{i, r} : \ldots : \hat{p}_0^{i, K} \approx \hat{p}_r^{i, r} : \hat{p}_0^{2r} : \ldots : \hat{p}_0^{Kr}
\]

for all repairs \( (r = 1 \ldots R) \).

This assumption also significantly reduces dimensionality of the problem, as now only \( R \) scaling factors \( X_{i+1} = \{X_{i+1}^1, X_{i+1}^2, \ldots, X_{i+1}^R\} \) must be found to get all new probabilities for the model \( M_{i+1} \):

\[
p_{i+1}^{r,s} = X_{i+1}^r \cdot p_0^{sr}, \quad r = 1 \ldots R, \quad s = 1 \ldots K
\]

Moreover, although frequency of a repair \( r \) depends on probabilities of all repairs (modifying probability of one repair changes, among others, state durations in the whole model, thus it changes frequency of all states) it can be assumed that in case of a single-step small adjustment its dependence on repairs other than \( r \) can be considered negligible and \( F'_r = F'_i(X_r, X_i^2, \ldots X_i^K) \approx F'_i(X_r^i) \). With these assumptions generation of a new model is reduced to the problem of solving \( R \) nonlinear equations in the form of \( F'_i(X_r^i) = F'_G \). This can be accomplished with one of standard root-finding algorithms.

Development described in this paper has been implemented and verified on practical examples with the following three approximation algorithms: Newton method
working on linear approximation of $F'(r)$, the secant method and the false position (falsi) method.

(A) Newton method on linear approximation (NOLA)

In this solution it is assumed that $F'(r)$ is a linear function defined by points $F(r_i)$ (obtained after solving the model in step 1°) and $F(0)$ (which is zero). Then simply

$$r_{i+1} = \frac{F(r_i)}{F'(r_i)}$$

Noteworthy advantage of this approach lies in the fact that no other point than the current frequency $r_i$ is required to compute the next approximation, so errors of previous steps do not accumulate and convergence is good from the first iteration.

(B) The secant method

In this standard technique the function is approximated by the secant defined by the last two approximations in points $r_i$, $r_{i+1}$ and a new one is computed as:

$$X'_{i+1} = X_i' - \frac{X_i' - X_{i-1}'}{F'(r_i) - F'(r_{i-1})}$$

After that $X_{i-1}'$ is discarded and $X'_{i+1}$ and $X_i'$ are considered in the next iteration.

To begin the procedure two initial points are needed. In this method the first point is chosen as the initial frequency of the model $M_0$ ($X_0'=1$), while the second point is computed as in NOLA method above: $X_i' = \frac{F(r_i)}{F'(r_i)}$.

(C) The false position (falsi) method

In this approach $X_{i+1}'$ is computed as in (3) but the difference lies in choosing points for the next iteration. While in (B) always $X_{i-1}'$ is dropped, now $X_{i+1}'$ is paired with that one of $X_i'$, $X_{i-1}'$ which lies on the opposite side of the root. In this way when (3) is applied the solution is bracketed between $X_i'$ and $X_{i+1}'$ (which is the essence of falsi method).

As in (B), the two initial points are needed but now they must lie on both sides of the root, i.e.

$$(F(r_0') - F(r_0')) (F(r_i') - F(r_{i-1}')) < 0$$

Choosing such points poses some difficulty. To avoid multiple sampling, it is proposed to select $X_0'=1$ (as previously) and then to compute $X_i'$ like in NOLA method but with some "overshoot" that would guarantee (4):

$$X_i' = \left(\frac{F(r_i')}{F(r_i')}\right)$$

with parameter $\alpha > 1$ controlling the overshoot effect. The overshoot must be sufficient to ensure (4) but, on the other hand, should not produce too much of an error as this would deteriorate approximation process during initial steps and would produce extra

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Figure 3. Life curve of equipment for some default maintenance policy (1) and life curves generated form Markov models adjusted to modified policies (2-5).
iterations of the method. If (4) is not met by initial value of $X_i$ (5) can be re-applied with increased value of $\alpha$, although it should be noted that each such correction requires solving a new $M_1$ model and in effect this is the extra computational cost almost equal to that of the whole iteration.

### 3.3 Comparison of the methods

The Markov model discussed here as an example has 3 deterioration states and 3 repairs ($K = R = 3$) with Mx1 representing minor, Mx2 medium and Mx3 major repairs. The life curve estimated from model $M_0$ is shown in Figure 3 as case (1). Cases (2) to (5) were created through adjusting $M_0$ to modified maintenance policies as follows:

- **case (2)** – frequencies of all repairs were reduced to 50% ($F_G = \frac{1}{2}F_0$)
- **case (3)** – all repairs but major (Mx3) were removed ($F_G = [0, 0, F_0^3]$)
- **case (4)** – frequencies of all repairs were reduced to 25% ($F_G = \frac{1}{4}F_0$)
- **case (5)** – all repairs were removed ($F_G = [0, 0, 0]$).

All three approximation methods (NOLA, secant and falsi) converged properly to the probabilities that give desired goal frequencies. Figure 4 presents details regarding the convergence process during computations of cases (2) ÷ (4).

Comparing the effectiveness of the methods it should be noted that although simplifications of the NOLA solution may seem critical, in practice it works quite well. As it was noted before, this method has one advantage over its more sophisticated rivals: since it does not depend on previous approximations, selection of the starting point is not so important and the accuracy during the first iterations is often better than in the secant or falsi methods. In the example in Figure 4 a NOLA method reached accuracy of 4.4% already after 2 iterations, while for secant and falsi methods the errors after two iterations were, respectively, 11% and 9.2%. Superiority of the latter methods, especially
of the falsi algorithm, manifests itself in the later stages of approximation when the potential problems with an initial selection of the starting points have been diminished.

4. Asset Risk Manager
The Asset Risk Manager (ARM) is a software package which uses the concept of a life curve and discounted cost to study the effect of equipment aging under different hypothetical maintenance strategies ([8]). The curves generated by the program are based on complex Markov models that were discussed in the two previous sections. For the program to generate automatically the life curves, default Markov model for the equipment has to be built and stored in the computer database. This is done through the prior running of the AMP program. Therefore, both AMP and ARM programs are closely related, and usually, should be run consecutively.

Implementation details of Markov models, tuning its parameters and all other internal details should not be visible to the non-expert end user. All final results are visualized either through easy to comprehend concept of a life curve or through other well-known concepts of financial analysis. Still, prior to running the analysis some expert involvement is needed, largely in preparation, importing and adjusting AMP models.

4.1 User input
A typical study is described through a comprehensive set of parameters which fall into three broad categories.

(1) General data. The Markov model of the equipment in question and its current state of deterioration form the primary information that is the starting point to most of ARM computations. The Markov model represents the equipment with present maintenance policy and is selected from a database of imported AMP models which needs to be prepared by an expert in advance. Deterioration state, referred to as “Asset Condition” (AC) throughout the ARM, must be supplied by the end user as percentage of “as-new” condition. Besides, a number of additional general parameters need to be specified, such as the time horizon over which the analysis will be performed, discount and inflation rates for financial calculations etc.

(2) Present maintenance policy. It is assumed that three types of maintenance repairs can be performed: minor, medium and major. These correspond to appropriate states in Markov model and not all of them must be actually present in the policy. For each repair user supplies its basic attributes, e.g. cost, duration and frequency.

(3) List of alternative actions. These are the hypothetical maintenance policies that decision-maker can chose from. Each action is defined as one of four types:
- continue as before (i.e. do not change the present policy),
- do nothing (i.e. stop all the repairs),
- refurbish,
- replace,

Apart from the first type, every action can be delayed for a defined amount of time. Additionally, for “non-empty” actions (i.e. any of the last two types) user must specify what to do in the period after action; choices are: (a) to change type of equipment and / or (b) to change maintenance policy. For every action user must also specify what to do in case of failure: whether to repair or replace failed equipment, its condition afterwards, cost of this operation etc. Thanks to these options a broad range of maintenance situations can be modeled.

The first action on the list is always “Continue as before” and this is the base of reference for all the others. The ARM can be directed to compute life curves, cost curves, or probabilities of failure – for each action independently – and then to visualize computed data in many graphical forms to assist the decision-maker in effective action assessment.

It should be noted that while the need for some action (e.g., overhaul or change in maintenance policy) is identified at the present moment, the actual implementation will usually take place only after a certain delay during which the original maintenance policy is in effect. Using ARM it is possible to analyze effect of that delay on the cost and reliability parameters.
4.2 Life curves
As it has been pointed out before, computing the average first passage time (FPT) from the first deterioration state ($D1$) to the failure state ($F$) in the Markov model yields an average lifetime of the equipment, i.e., length of its life curve. On the other hand, solving the model for state probabilities of all consecutive deterioration states makes possible computing state durations, which in turns determine shape of the curve. Simple life curves obtained for different maintenance policies are later combined in constructing composite life curves which describe various maintenance scenarios.

For sake of simplicity and consistency, always exactly three deterioration states, or levels, are presented to the end user: minor, medium and major, with adjustable AC ranges. In case of Markov models which have more than three $D$s states, the expert decides how to assign Markov states to the three levels when importing the model.

Figure 5 shows exemplary life curves computed for typical maintenance situations. In each case the action is delayed for 3 time units (months, for example) and the analysis is performed for a time horizon of 10 t.u.. In case of failure seen in “Do nothing” action, equipment is repaired and its condition is restored to 85%.

4.3 Probability of failure
For a specific action, probability of failure within the time horizon ($PoF_{TH}$) is a sum of two probabilities: of failure taking place before ($PoF_B$) and after ($PoF_A$) the moment of action. It is assumed that failures in these two periods making up the time horizon are independent, so $PoF_{TH} = PoF_B + PoF_A - PoF_B \cdot PoF_A$.

To compute $PoF$ within given time period ($T$), the Markov model for the equipment and the life curve are required. The procedure is as follows:

1. For initial asset condition, find from the life curve the current deterioration state $DS_n$; compute also state progress $SP$ (%), i.e. estimate how long the equipment has been in the $DS_n$ state.
2. Running FPT analysis on the model, find distributions $D_n$ and $D_{n+1}$ of first passage time from $DS_n$ and $DS_{n+1}$ to the failure state $F$.
3. Taking state progress into account, probability of failure is evaluated as $PoF = D_n(T) \cdot (1 - SP) + D_{n+1}(T) \cdot SP$.

For better visualization, rather than finding a single $PoF_{TH}$ value for action defined by the user in input parameters, ARM computes a curve which shows the $PoF_{TH}$ as a function of action delay varying in a range $0 \div 200\%$ of user-specified initial value. An example is demonstrated in Figure 6 for “Do nothing”
4.4 Cost curves
In many financial evaluations, the costs are expressed as present value (PV) quantities. The present value approach is also used in ARM because maintenance decisions on aging equipment include timing, and the time value of money is an important consideration in any decision analysis. The cost difference is often referred to as the Net Present Value (NPV). In the case of maintenance, the NPV can be obtained for several re-investment options which are compared with “Continue as before” policy.

Cost computations involve calculation of the following cost components:
1. cost of maintenance activities,
2. cost of the action selected (refurbishment or replacement),
3. cost associated with failures (cost of repairs, system cost, penalties).

To compute the PV, inflation and discount rates are required for a specified time horizon. The cost of maintenance over the time horizon is the sum of the maintenance costs incurred by the original maintenance policy for the duration of the delay period, and the costs incurred by the new policy for the remainder of the time horizon. The costs associated with equipment failure over the time horizon can be computed similarly except that the failure costs before and after the action must be multiplied by the respective probabilities of failures (PoF\textsubscript{B} and PoF\textsubscript{A}), and the two products added.

As in case of probability of failure, ARM presents the end user with a curve which shows the cost as a function of action delay varying in a range 0 ÷ 200% of user-specified value.

5. Conclusions
The purpose of ARM tool is to help in choosing effective maintenance policy. Based on Markov models representing deterioration process, the equipment life curve and other reliability parameters can be evaluated. Once a database of equipment models is prepared, the end-user can perform various studies about different maintenance strategies and compare expected outcomes. As the results are visualized through the relatively simple concept of a life curve, no detailed expert knowledge about internal reliability parameters or configuration is required.

References:


