Histogram-Based Optimal Multiple Thresholding Using Genetic Algorithm

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ABSTRACT

A novel approach for truly segmenting gray scaled images through thresholding is presented. Thresholding is considered as an optimization problem. Genetic algorithm is used to search for the optimal thresholds. Single and multiple optimal thresholds are considered. Results showed that genetic algorithm is very promising in the area of image segmentation.

Key Words: Image Segmentation, Optimal Thresholding, Genetic Algorithm, Fitness Function.

1. Introduction

Image segmentation is an important step in many computer vision applications. Image segmentation algorithms are generally based on one of two basic properties of gray-level values: discontinuity and similarity. In the first category, the approach is to partition an image based on abrupt changes in gray level. The principle areas of interest within this category are detection of lines and edges in an image. The principal approaches in second category are based on thresholding, region growing, and region splitting and merging. Different segmentation may be obtained from conventional segmentation techniques. Since almost every image segmentation algorithm has parameters to control the segmentation, there are no general algorithms, which will work for all images. [1]

Image thresholding is widely used for image segmentation. It can be classified as bi-level thresholding and multilevel thresholding. Bi-level thresholding classifies the pixels into two groups, one including those pixels with gray levels above a certain threshold, the other including the rest. Multilevel thresholding divides the pixels into several groups; the pixels of the same group have gray levels within a specific range.

Over the years many thresholding techniques have been proposed [6,13]. They can be classified into two types: optimal thresholding methods and property-based thresholding methods [1]. Optimal thresholding methods search the optimal thresholds which make the thresholded classes on the histogram reach the desired characteristics. Usually, it is made by optimizing an objective function.

In this research paper, thresholding is dealt with as a combinatorial optimization problem. A novel formulation and implementation of a general randomized search approach to threshold images, using genetic algorithms, based on image histogram; is presented in this paper.

2. Image Segmentation Based on Thresholding

In digital image processing, thresholding is a well-known technique for segmenting gray level images. Because of its wide applicability to other areas of digital image processing, a variety of techniques have been proposed over the years for determining thresholds at which to segment an image in order to extract objects from their background [2]. ***Thresholding*** in its simplest form means; to classify the pixels of a given image into two regions (e.g., objects and background); one including those pixels with their gray values above a certain threshold. [3].

Thresholding techniques are divided mainly into two types; global and local-techniques [2,4]. A global thresholding technique is one that thresholds the entire image with a single threshold value, whereas a local thresholding technique is one that partitions a given image into subimages and determines a threshold for each of these sub images. A simple and general approach to determine the threshold value 'T' is by calculating the global average of the image. In local thresholding techniques, the above operation is applied on each sub image, instead of the whole image.

However; those boundaries; which correspond to object-object discontinuities, or to internal structure of the object, may not exhibit a clear foreground-background distinction (such as images of natural outdoor scenes). One cannot expect a single threshold to detect all or even most of the object boundaries in the scene. In order to detect most of the interesting boundaries within an image, some alternative or additional processing must be done to obtain a clearer result [5]. One alternative is to use multiple thresholds [6] or threshold that varies across an image [7]. These multiple thresholds will be defined in terms of a set of n thresholds rather than a single threshold. These *n* thresholds will partition the feature space into *n + 1* possible classes.

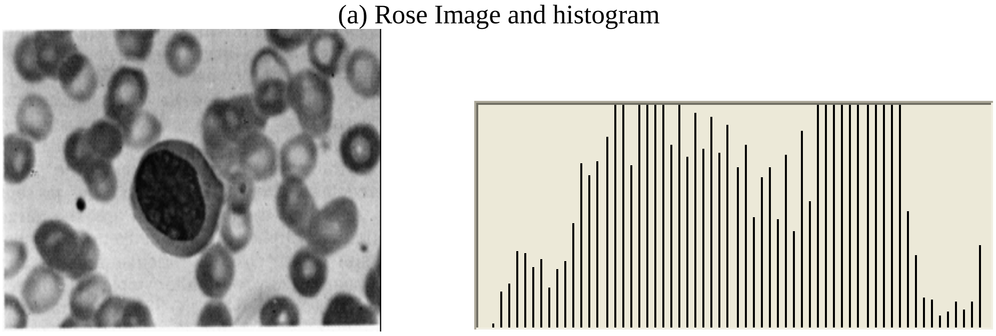
Unless the object in the image has extremely steep sides, the exact value of threshold gray level can have considerable effect on the boundary position and overall size measurement, particularly the area measurement, which is sensitive to the threshold gray level. For this reason, an optimal, or at least consistent, method to establish the threshold is needed [8].

In the following sections a genetic based optimal single and multiple, global thresholding method is introduced.

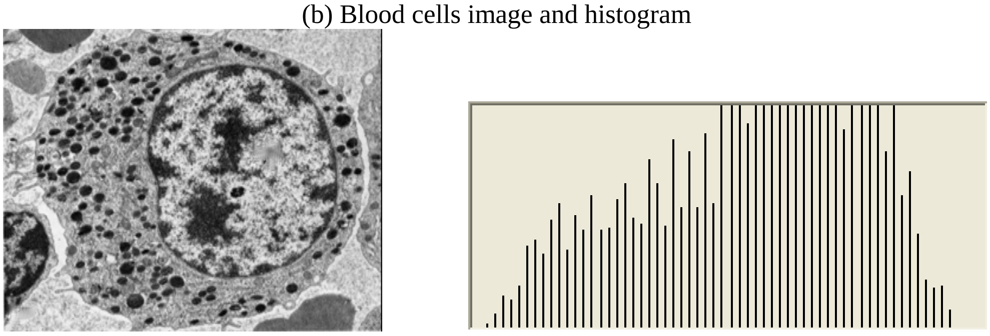
3. Genetic Algorithm Search

Genetic Algorithms (GAs) are a class of iterative procedures that simulate the evolution process of a population of structures subject to the competitive forces prescribed in ‘survival of the fittest’ principle. The process of evolution is random yet guided by a selection mechanism based on the fitness of individual structures. If carefully designed and properly implemented, a GA will exhibit a behavior similar to that described in evolution theory- relatively high fitness structures have a larger chance to survive and to produce even higher fitness offspring. The result will be an increase in the overall fitness of a population in each generation [9].

(a) Rose Image and histogram



(b) Blood cells image and histogram



(c) Blood cell image and histogram

Figure 1: 256 gray level images used as input to the
Global Optimal Multiple thresholding System

GAs start by randomly generating a population of individuals (chromosomes). Each represents a point in the search space. The chromosomes are then evaluated to obtain a quantitative measure of how well they perform as possible problem solutions. Reproductive opportunities are allocated such that the best individuals receive more opportunities to reproduce than those which have poor performance; this bias need not be great to produce the required selective pressure to allow “artificial selection” to occur. This search uses some probabilistic calculations to find parameter sets and tends to be slower but has greater success at finding the global optima.

To apply a genetic algorithm to any practical problem, it requires:

1. A structural (chromosomal) representation of solution to the problem.
2. An evaluation (objective function) of individuals in terms of their “fitness”.
3. A method to initialize the population of candidate solutions.
4. Values of parameters which the genetic algorithm uses (e.g., population size, crossover rate. etc.).
5. Genetic operators which produce new sets of individuals.
6. Selecting mating pool for next generation.
7. Termination criteria for the genetic algorithm.

4. Global Optimal Multiple thresholding System

A Genetic Algorithm based Global Optimal Multiple image Thresholding (GOMT) system is developed and implemented. Both optimal single image thresholding (SOT) as well as optimal multiple image thresholding (MOT) are considered. Searching for a single optimal threshold is an NP-complete problem. Searching for optimum multiple threshold increases the complexity of the problem. It is the effectiveness and high convergence rate of the genetic algorithms that encouraged the authors to consider MOT. Different components of the GOMTsystem are described hereafter.

4.1 Chromosomal Representation and Selection

An individual (chromosome) is an array of parameter values (genes) to be optimized. If the individual has N_{gen} given by $g_{1,}$ g_2, ……, g_{Ngen} then the individual is written as an array with 1x N_{gen} elements so that; Individual = [g_1, g_2,….., g_{Ngen}].

In searching for a single optimum threshold (SOT), the solution is coded as binary number that can take any value between 0 and L-1; were L is the number of gray levels in the image. Each individual is simply a finite-length of 8-bits (1’s and 0’s), which represent the coded string between 0 (00000000) to code L-1 (generally 255 i.e.11111111). The number 0 represents the lower bound (black level), where the number L-1 represents the upper bound (white level).

Parent$_1$	10	30	120	180	220
Parent$_2$	***25***	***80***	***220***	***30***	***140***

offspring$_1$	***10***	*180*	***120***	*30*	*140*
offspring$_2$	***25***	***80***	*140*	*180*	*220*

(a) PMX crossover operator

Parent$_1$	10	20	70	90	50
Parent$_2$	90	30	180	80	20

offspring$_1$	10	**30**	70	90	**20**
offspring$_2$	90	**20**	180	80	**50**

(b) CX crossover operator

Chromosome	6	1	5	3	2	4

Chromosome	6	**2**	5	3	**1**	4

(c) Mutation operator

Figure 2: Crossover and mutation operators as used in the *Global Optimal Multiple thresholding System* (Numbers represent threshold points)

In searching for a multiple optimum thresholds (MOT), each gene of the individual is a threshold point and is represented by an integer number within the interval [0, L-1]. The individual -in this case- has number of genes N_{gen} equal to the number of thresholds or the number of segmentation regions -1.

An evaluation or fitness function has to be formulated to discriminate between the different individuals as a measure of the properness of the threshold value(s) obtained. Such evaluation function will be derived later.

To start the search the GOMT generates an ***initial population*** of individuals randomly (i.e. using random 8 bits binary number generator the in case of SOT and random decimal integer number generator in the range [0, L-1] in the case of MOT).

Mate ***selection*** and pairing is performed using the roulette wheel selection technique; where each current chromosome in the population has a roulette wheel slot sized in proportion to its fitness [10]. A target value is set, which is a random proportion of the sum of the fitness in the population. Two chromosomes are selected from the mating pool of good chromosomes to produce two new offsprings. Uniform random number generator is used to select chromosomes.

4.2 Crossover and Mutation

Mating is the creation of one or more offspring from the parents selected in the pairing process. The crossover operator is repeatedly applied to pairs of individuals [11]. The conventional simple crossover operator is used in SOT. Such crossover failed in producing valid solutions in the case of MOT due to threshold point repetitions on the same individual. PMX and CX crossover [10] operators were used in MOT. System performance in each case is discussed later.

Mutation operator assures that the system avoids the problem of overly fast convergence that may end up in a local rather than a global minimum. It forces the routine to explore other areas of the cost by randomly introducing changes, or mutation, in some of the genes. Simple mutation is used with binary strings (SOT). Such mutation does not work well in MOT; swap mutation [10] is used instead.

4.3 Termination

The termination criteria used for bringing the search to a halt is either of the following;

- When the maximum value of the fitness function in the entire population is equal to the average fitness value (all individuals have identical fitness).
- There is always a limit to the maximum number of generations. Reaching this limit would indicate the best solution.

5. Calculating Fitness Function

Ideally; if the image consists of objects on a background, the histogram has a deep and sharp valley between two peaks representing objects and background, respectively. Threshold in such case can be chosen at the bottom of the valley. However, for most real images, it is often difficult to detect the valley bottom precisely, especially in such cases as when the valley is flat and broad, imbued with noise, or when the two peaks are extremely unequal in height, often producing no traceable valley.

The valley sharpened technique [2,12] was proposed to overcome these difficulties. In this method, selecting threshold depends only on the gray level histogram without other prior knowledge and is based on discrimination analysis that maximizes class separation.

Suppose the pixels of a given picture be represented in L gray level [0,1,….,L-1]. The number of pixels at level i is denoted by n_i and the total number of pixels by $N = n_0 + n_1 + \dots + n_{L-1}$

In order to simplify the discussion, the gray level histogram is normalized representing the image probability distribution:

$$p_i = \frac{n_i}{N}, \ p_i \ge 0, \ \sum p_i = 1 \qquad ...(1)$$

Now threshold operation is partitioning of image pixels into two classes C_0 and C_1 (background and object, or vice versa) at gray level t; C_0 denotes pixels with level [0,1,…..,t] and C_1 denotes pixels with level [t+1,t+2,...,L-1]. Then the probabilities of class occurrence and the class mean level respectively, are given by

$$w_0 = \Pr(C_0) = \sum_{i=0}^{t} p_i \qquad ...(2)$$

$$w_1 = \Pr(C_1) = \sum_{i=t+1}^{L-1} p_i = 1 - w_0 \qquad ...(3)$$

$$\mu_0 = \frac{\mu_t}{w_0} \qquad ...(4)$$

$$\mu_1 = \frac{\mu_T - \mu_t}{w_1} \qquad ...(5)$$

where

$$\mu_t = \sum_{i=0}^{t} i p_i \qquad ...(6)$$

is the mean gray value up to level t and

$$\mu_T = \sum_{i=0}^{L-1} i p_i \qquad ...(7)$$

is the mean gray value over the entire image.

In order to evaluate the “goodness” (fitness function) of the threshold at level t, the measure of class separability used in the discriminate analysis is defined as:

$$\lambda = \frac{\sigma_B^2}{\sigma_T^2} \qquad ...(8)$$

where

$$\sigma_B^2 = w_0 w_1 (\mu_1 - \mu_0)^2 \qquad ...(9)$$

$$\sigma_T^2 = \sum_{i=0}^{L-1} (i - \mu_T)^2 p_i \qquad ...(10)$$

$\sigma_B{}^2$ and $\sigma_T{}^2$ are between-class variance and total class variance of levels, respectively. Then the problem is reduced to an optimization problem to search for a threshold *t* that maximizes the object function of equation (8). This standpoint is motivated by a conjecture that good threshold classes would be separated in gray levels, and conversely, a threshold giving the best separation of classes in gray levels would be the best threshold.

The fitness function for the SOT is that of equation (8). The lower bound (zero) is attainable by, and only by, fitness having a single constant gray level, and the upper bound (unity) is attainable by, and only by, two-value images. The result of this method has two- peaked histogram. Also, this method can work successfully in determining the threshold value even if the histogram has no clear valley.

Table 1: GOMT setup of SOT for the images of figure 3.

Population size	10 Individuals
Chromosome Length	8 (Binary string)
Selection Operator	Roulette Wheel
Crossover operators	Simple (single-point)
Mutation Operator	Complement method
Replacement	By-rank
Crossover rate	0.90
Mutation rate	0.080

Table 2: Results from SOT search.

Image	Initial Fitness	Final Fitness	Solution generation	SOT
Rose	0.776968	0.787586	25	108
Blood	0.700873	0.776322	14	135

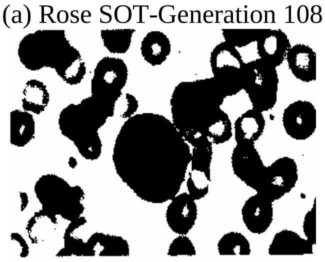
The extension of this method to multiple thresholding is straightforward. Class variance σ_B^2 in equation (8) has to be changed to σ_{Bm}^2:

$$\sigma_{Bm}^2 = w_0(\mu_0 - \mu_t)^2 + ... + w_{m-1}(\mu_{m-1} - \mu_t)^2 \quad ...(11)$$

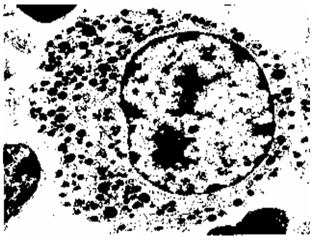
6. Experimental Results and Discussion

Results of applying SOT and MOT on two images of size 200x320 pixels and 256 gray levels are reported. Single optimal threshold for ‘rose’, ‘blood cells’ and ‘blood cell’ images are shown in figure 3. Optimal threshold is obtained after 108, 135, and 125 generations respectively. The GOMT setup is given in table 1. Table 2 gives the numerical results of single optimal threshold search. From table 2 and binary images of figure 3 one can see that the same genetic parameters were used and the rate of convergence toward the solution is image dependent. This may be expected since the threshold value is image dependent too. Clearly the objects within both (original) images are perfectly separated from background.

(a) Rose SOT-Generation 108



(b) Blood cells SOT-Generation 135



(c) Blood cell SOT- Generation 125

Figure 3: Single optimal threshold: 2 regions (binary).
(Convergence differ for different images)

Table 3: GOMT setup of MOT for the image of figure 4 and 5.

Population size	100 Individuals
Chromosome Length	4-5 (Decimal Integers)
Selection Operator	Roulette Wheel
Crossover operators	CX & PMX (single-point)
Mutation Operator	Swap mutation
Replacement	By-rank
Crossover rate	0.70 (PMX)
Mutation rate	0.05-0.10

Table 4: Results from MOT search for the image of figure 1a.

Crossover Method	CX	PMX
Crossover rate		0.7
Mutation Rate	0.1	0.05
Initial Fitness	0.964994	0.968808
Final Fitness	0.976463	0.976416
Solution generation	35	34
Optimal Multiple Thresholds	33,70,111,154,193	33,73,115,160,196

Table 5: Results from MOT search for the image of figure 1b.

Crossover Method	CX	PMX
Crossover rate		0.7
Mutation Rate	0.1	0.1
Initial Fitness	0.938524	0.927421
Final Fitness	0.948517	0.947454
Solution generation	27	20
Optimal Multiple Thresholds	61,103,146,184	69,109,149,186

Table 6: Results from MOT search for the image of figure 1a (Replacement: Week Parent).

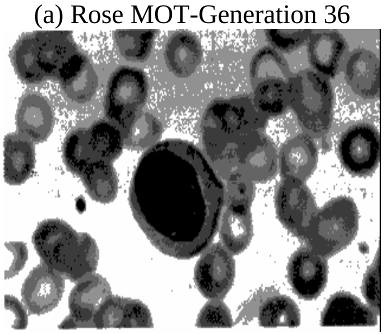
Crossover Method	CX	PMX
Crossover rate		0.7
Mutation Rate	0.85	0.06
Initial Fitness	0.966808	0.966808
Final Fitness	0.976143	0.976139
Solution generation	37	38
Optimal Multiple Thresholds	33,70,112,160,200	39,73,116,160,197

Table 7: Results from SOT search for the image of figure 1a. (Dynamic Population) .

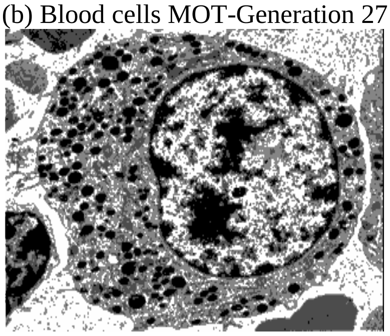
Population Size Rate	0.5
Initial Population	10
Final Population	14
Initial Fitness	0.776968
Final Fitness	0.787586
Solution generation	11
Optimal Multiple Thresholds	108

Table3 lists the setup data of the GOMT for segmenting same images of figure 1 using multiple thresholds. Different number of regions (thresholds) is considered. System performance using cyclic or partially matched crossover operators were investigated. Tables 4 and 5 list the results of multiple optimal thresholds search. The percentage difference in threshold values is very small when using either of the two crossover operators. Increasing the number of threshold complicates the problem and increases the number of generations required to reach the optimal solution. The system with partially matched crossover seems to be converging little bit faster than using the cycle crossover. Convergence rate still is image dependent. Images of figures 4 and 5 show the thresholded images obtained from the system using CX and PMX operatores. Thresholded images at intermediate generations (before reaching the solution) are obtained and compared but not given here to minimize paper size. No difference can be sensed from the final results obtained using either of the two used crossover operators.

(a) Rose MOT-Generation 36



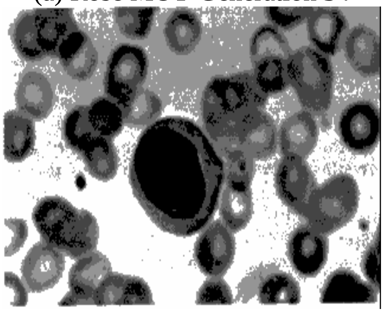
(b) Blood cells MOT-Generation 27



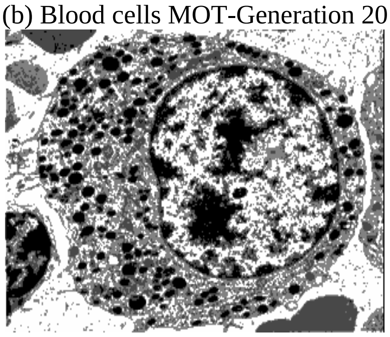
(c) Blood cell MOT- Generation 37

Figure 4: Multiple optimal thresholds: 6 regions. (Using CX operator)

(a) Rose MOT-Generation 34



(b) Blood cells MOT-Generation 20



(c) Blood cell MOT- Generation 22

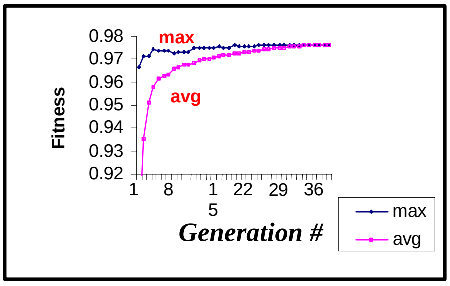
Figure 5: Multiple optimal thresholds: 6 regions. (Using PMX operator)

Table 6 represents the result of applying GA to ‘rose’ image with another method of replacement (weak parents). CX and PMX crossover operators showed different performance than that of by rank replacement. Figure 6 shows performance in case of weak parents for both CX and PMX operators. PMX convergence is faster. Comparing results of performance shown in figure 6 with results shown in figure 7 for same operators but by rank replacement shows that convergence is faster in by rank replacement.

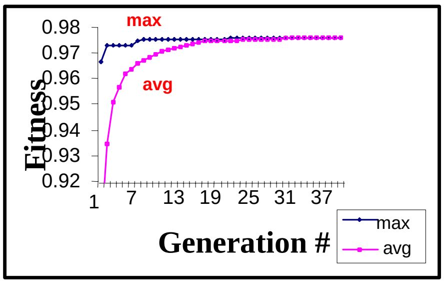
Dynamic population size implementation results are shown in table 7 for SOT search. A seed population of some size will always be necessary to start the search (0.7 in this case). The new chromosomes, which are generated using genetic operators, are appended with the old generation and population size is selected using rate called population rate. It is used in the same manner as crossover rate. The fast convergence is very clear from results shown.

7. Conclusion

A Genetic Algorithm based Global Optimal Multiple image Thresholding (GOMT) system was presented. The thresholding process was considered as an NP-complete problem. The GOMT system is able to find optimal single or multiple thresholds. GOMT performance showed that the genetic algorithm is promising in the area of image segmentation due to its ability to find true segmentation based on optimal thresholds in very short time as compared to traditional optimization methods. The paper studied the effect of different GA parameters on the performance of GOMT. The multiple optimal thresholding method implemented and tested in this paper may be used as an optimal solution for non-uniform image quantization.

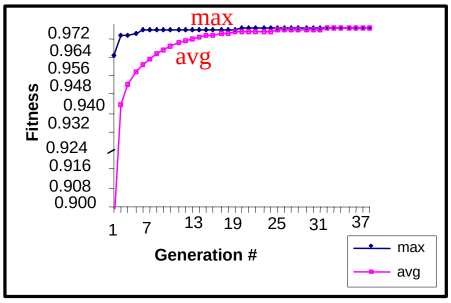


(a) Best of gen(max) result and average result to gen(37) (CX operator)

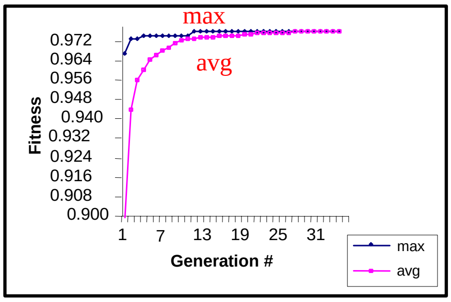


(b) Best of gen(max) result and average result to gen (38) (PMX Operator)

Figure 6: Relationship between fitness and average for 'rose' image using weak parents replacement



(a) Best of gen(max) result and (avg) result to gen(36) (CX operator)



(b) Best of gen(max) result and average result to gen (34) (PMX Operator)

Figure 7: Relationship between fitness and average for 'rose' image using by rank replacement

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