DISTRIBUTED CONSTRUCTION OF WEAKLY CONNECTED DOMINATING SETS FOR CLUSTERING MOBILE AD HOC NETWORKS

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Abstract: Weakly connected dominating set (WCDS) has been proposed to cluster mobile ad hoc networks and be used as a virtual backbone. There have been some distributed approximation algorithms proposed in the literature for minimum WCDS. But none of them have constant approximation factors. Thus these algorithms can not guarantee to generate a WCDS of small size. Their message complexities may also be as large as $O(n^2)$. In this paper, we design a new distributed algorithm that outperforms the existing algorithms. This algorithm has an approximation factor of at most 5 and linear message complexity. Our algorithm requires only single-hop neighborhood knowledge and a message length of $O(1)$. So it is practical.

Keywords: Mobile ad hoc networks, Dominating Sets, Weakly Connected Dominating Set, virtual backbone.

1. INTRODUCTION

Mobile ad hoc network is an autonomous system consisting of mobile hosts connected by wireless links. It can be flexibly and quickly deployed for many applications such as automated battlefield operations, search and rescue, and disaster relief. Unlike wired networks or cellular networks, there is no any physical infrastructure and central administration in mobile ad hoc networks. Every host can move to any direction at any speed and any time. This induces a dynamic topology. Due to the broadcast advantage of wireless communication, the transmission of one host can be heard by all hosts in its communication range. If two hosts are not located in each other's transmission range, intermediate relay hosts must be employed as bridges to build communication paths. This is the multihop characteristic of the mobile ad hoc network, for which routing decisions must be made for far-away hosts to communicate. Mobile ad hoc network has a very strict resource constraint. Wireless mobile are usually light-weight and battery-powered. Compared with wired lines, wireless links have much less available bandwidth.

These features make routing the most challenging problem in mobile ad hoc network. Existing routing protocols can be classified into two categories: proactive and reactive. One important observation on these protocols is that none of them can avoid the involvement of flooding. For example, proactive protocols rely on flooding for the dissemination of topology update packets; reactive protocols rely on flooding for route discovery. Flooding suffers from the notorious broadcast storm problem [1]. Broadcast storm problem refers to the fact that flooding may result in excessive redundancy, contention, and collision. This causes high protocol overload and interference to ongoing traffic.

Recently an approach based on overlaying a virtual infrastructure on an ad hoc network is proposed in [2]. Routing protocols are operated over this infrastructure, which is termed core. The key feature in this approach is the new core broadcast mechanism which use unicast to replace the flooding mechanism used by most on-demand routing protocols. The unicast of route request packets are restricted to core nodes and a (small) subset of non-core nodes. Simulation results indicate that the core structure is effective in enhancing the performance of the routing protocols. Actually prior to this work, inspired by the physical backbone in a wired network, many researchers proposed the concept of virtual backbone
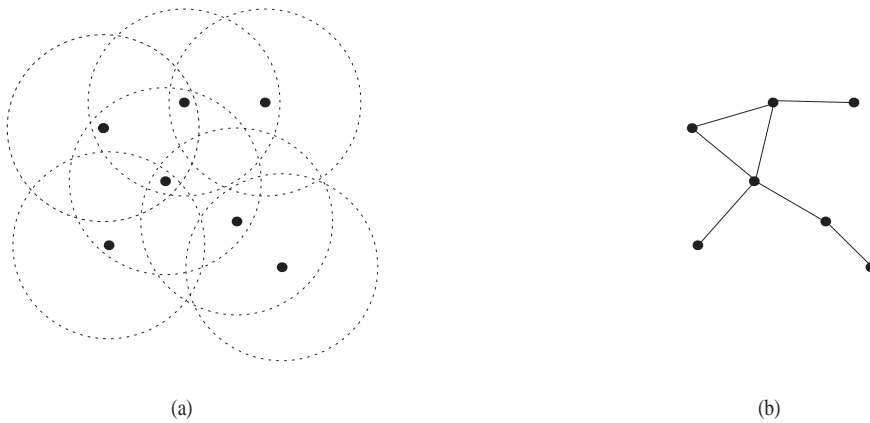
Figure 1 Model mobile ad hoc networks by unit disk graphs

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or unicast, multicast/broadcast in mobile ad hoc networks (see [3-6]). After that some related works have also been done (see [7-10]). In the specialized literature there is a large consensus on the fact that the backbone should be a dominating set, i.e., each node is either in the backbone or next to some node in it. Moreover, the following additional features are widely considered to be appealing: (i) the backbone should be "small", (ii) it should be connected or weakly connected, and (iii) it should be constructed with low communication and computation costs.

In this paper, we study the problem of efficiently constructing a weakly connected dominating set (WCDS in short) in mobile ad hoc networks. This problem was firstly studied by Chen and Liestman in [10]. They proposed three distributed algorithms. All of them have a logarithmic approximation factor. Thus none can guarantee to generate a WCDS of small size. These algorithms also have very high implementation cost in terms of message complexity. A new distributed algorithm is proposed in this paper. The size of the WCDS delivered by our algorithm is at most 5 times of the optimal solution. Its message complexity is linear. In terms of approximation factor and message complexity, our algorithm outperforms the existing ones. Moreover our algorithm requires only single-hop neighborhood knowledge and a message length of $O(1)$. So it is practical to be implemented.

The remaining of the paper is organized as follows. Section 2 introduces the network model and related works. In section 3, we propose our algorithm. Finally we conclude this paper in section 4.

2. PRELIMINARIES

2.1 Network model

In this paper, we assume a given mobile ad hoc network instance contains n nodes. Each node is in the ground and is mounted by an omni-directional antenna. Thus the transmission range of a node is a disk. We further assume that each transceiver has the same communication range. Thus it can be modeled as a unit disk graph [11], a geometric graph in which there is an edge between two nodes if and only if their distance is at most one. An example is shown in Figure 1. In the figure, (a) is a mobile ad hoc network with 7 nodes, where dotted circles represent the communication range. (b) is the corresponding unit disk graph.

Given graph $G = (V, E)$, a dominating set (DS) is a node subset $S \subseteq V$, such that every node $v \in V$ is either in S or adjacent to at least one node in S. For any edge $(u, v) \in E$, if $u \in S$ and $v \notin S$, then u is v's dominator and v is u's dominatee. If the induced subgraph of S is connected, then S is a connected dominating set (CDS). Define the subgraph

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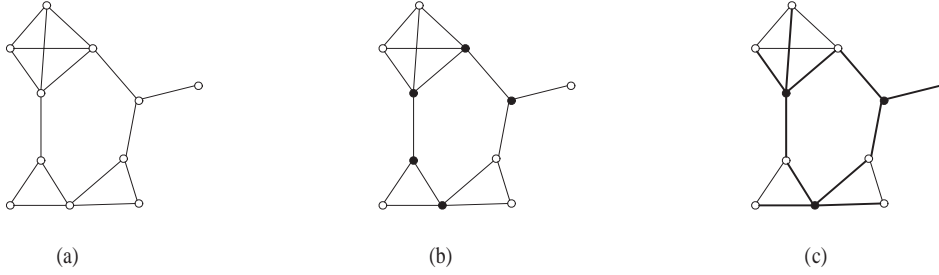
(a)   (b)   (c)

Figure 2 CDS and WCDS

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weakly induced by S as $\langle S \rangle_w = (N[S], E \cap (N[S] \times S))$, where $N[S]$ includes all nodes in S and their neighbors. The edges of $\langle S \rangle_w$ are all edges of G which have at least one end point in S. A node subset S is a weakly connected dominating set (WCDS), if S is a dominating set and $\langle S \rangle_w$ is connected. Figure 2 shows an example. (a) is a graph. In (b) those black nodes compose a CDS. In (c) those black nodes compose a WCDS, while the bold edges show the structure of $\langle S \rangle_w$.

Two nodes are independent if they are not neighbors. An independent set S of G is a subset of V such that for $\forall u, v \in S, (u,v) \notin E$. S is maximal if any node not in S has a neighbor in S. Any maximal independent set is also a dominating set.

2.2 Related works

In general graph case, and even in unit disk graph, the problem to find a DS/CDS/WCDS with minimum cardinality is NP-hard [11,12]. Thus only distributed approximation algorithms in polynomial time are practical for mobile ad hoc networks.

In many papers (see [3,6-9]), CDS is used as the virtual backbone in mobile ad hoc networks. Many distributed algorithms have been proposed to construct a CDS with small size. In [10], WCDS is proposed to be used as the virtual backbone. The reason is that in general a WCDS can be smaller than a CDS, which also means a smaller backbone. The authors firstly designed two approximation algorithms, which are based on the algorithms for CDS proposed by Guha and Khuller in [13]. Then they implemented the two algorithms distributedly to get two distributed algorithms. Finally a fully distributed algorithm was presented. The authors also analyzed the performance of the first two algorithms in general graph case, where the approximation factor of both algorithms are $\Theta(\log n)$.

We have analyzed the three algorithms of [10] in unit disk graph. Because of the limit of space, we omit the detail here and just list the results in table 1.

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Table 1 Performance comparison

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	[10]-I	[10]-II	[10]-III	Our algorithm
Approximation factor	$\Theta(\log n)$	$\Theta(\log n)$	$\Omega(\log n)$	5
Message complexity	$O(n^2)$	$O(n^2)$	$O(n^2)$	$O(n)$

3. OUR ALGORITHM

In this section, we present our distributed algorithm. We assume each node has an unique id and knows all of its neighbors and their ids. Such kind of information can be achieved by periodic beacons. We designate a node as the leader. If it is impossible to specify any leader, a distributed leader-election algorithm [14] can be applied. The network is also assumed to be synchronous, which means communication proceeds in synchronous rounds:
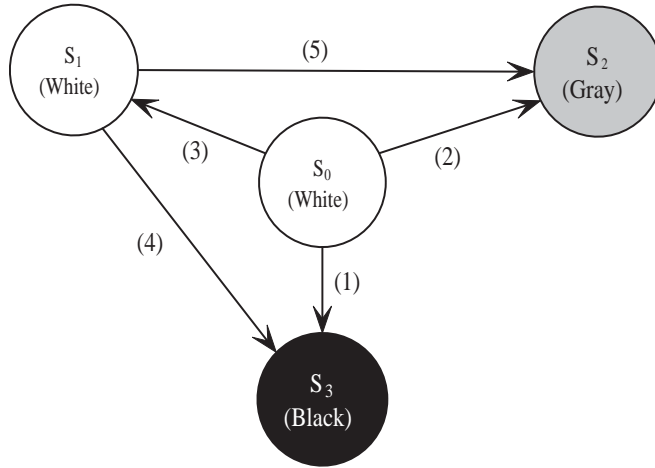
Figure 3 State transition diagram

in each round, every node sends messages to its neighbors, receives messages from its neighbors, and does some local computation.

At any time of the algorithm, each node can be in one of the four states: S_0, S_1, S_2 and S_3. S_0 is the initial state. A node in this state has white color (*note: Color is not necessary in our algorithm. It is retained in the algorithm description for the purpose of better easier elaboration.*). At the beginning all nodes are in S_0. S_1 is the active state. S_2 is the dominatee state. A node in this state is a dominatee and has gray color. S_3 is the dominator state. A node in this state is a dominator and has black color. At the end of the algorithm, all nodes in S_3 form the WCDS. Each node has a copy of the algorithm and begins to run it at the same round. The procedure of the algorithm is described as follows:

(1)   If node u is the leader, it declares itself as a dominator by broadcasting a DOMINATOR message, and goes to state S_3.

(2)   If node u is in state S_0 and receives a DOMINATOR message, it declares itself as a dominatee by broadcasting a DOMINATEE message, and goes to state S_2.

(3)   If node u is in state S_0 and receives a DOMINATEE message, it broadcasts an ACTIVE message and goes to state S_1.

(4)   If node u is in state S_1 and has the lowest id among all its active neighbors or there is no

active neighbor, it declares itself as a dominator by broadcasting a DOMINATOR message, and goes to state S_3.

(5)   If node u is in state S_1 and has bigger id than one of its active neighbor, it declares itself as a dominatee by broadcasting a DOMINATEE message, and goes to state S_2.

The state transition diagram of the algorithm is shown in Figure 3. Each arc corresponds to a step of the algorithm.

Figure 4 gives an example of the algorithm. The numbers labeled beside the nodes are their ids. Assume at the beginning node 0 is elected as the leader. For this example, 6 rounds are needed to compute the WCDS. The detail is described as follows:

Round 1: Node 0 marks itself black and sends out a DOMINATOR message(see Figure 4(a)).

Round 2: Upon receiving the DOMINATOR message from node 0, node 1 and 2 mark themselves gray, and send out the DOMINATEE messages(see Figure 4(b)).

Round 3: Upon receiving the DOMINATEE messages from node 1 and 2, node 3, 4 and 5 send out the ACTIVE messages and go to state S_1.
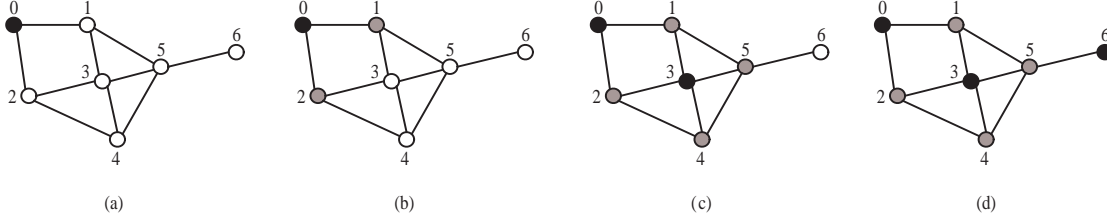
Figure 4 An example of the algorithm

Round 4: Node 3 has the lowest id among its active neighbor, so marks itself black and sends out a DOMINATOR message. Node 4 and 5 mark themselves gray and send out the DOMINATEE messages(see Figure 4(c)).

Round 5: Upon receiving the DOMINATEE message from node 5, node 6 sends out an ACTIVE message and goes to state S_1.

Round 6: Since node 6 has no active neighbor, so it marks itself black and sends out a DOMINATOR message (see Figure 4(d)).

Now we analyze the performance of the algorithm.

Theorem 1. *At the end of the algorithm, all the black nodes form a WCDS. The size of it is at most 5 times of the minimum WCDS.*

Proof. Let S be the set of the black nodes at the end of the algorithm. We prove that S is a maximal independent set. According to the algorithm, for each black node, all of its neighbor will be marked gray. So there are no two black nodes adjacent to each other. Moreover for each gray node, there is at least one black node adjacent to it. Otherwise the node can't be marked gray. So S is a maximal independent set, and is also a DS.

Now we prove S is a WCDS. To do that, we modify the algorithm minorly as follows:

- In step (2), when a node u changes its state from S_0 to S_2, let $f(u)$ record the id of the DOMINATOR message it receives at that time. If there are more than one DOMINATOR messages, let $f(u)$ be the smallest id among them.

- In step (3), when a node u changes its state from S_0 to S_1, let $f(u)$ record the smallest id among all the DOMINATEE messages it receives at that time.

- In step (6), when a node u changes its state from S_1 to S_2, let $f(u)$ record the smallest id among all the active neighbors.

Such modifications will not affect the result of the algorithm. At the end, each node u except for the leader

node has a value $f(u)$, by which we can construct a tree rooted at the leader node. For each gray node u, $f(u)$ must be a black node, while for each black node u, $f(u)$ must be a gray node. So this tree is a subgraph of $\langle S \rangle_w$, which means $\langle S \rangle_w$ is connected. Thus S is a WCDS.

Let OPT be the minimum WCDS and $v_1, v_2, \cdots v_k$ be the dominators in OPT. For $1 \le i \le k$, define S_i be the set of nodes in S dominated by node v_i. Since OPT is a DS, so we have

$$S_1 \cup S_2 \cup \cdots \cup S_k = S \qquad (1)$$

On the other hand, from the Theorem 3.1 of [15], in a unit disk graph, any node can dominate at most 5 independent nodes. Since S is a maximal independent set, so

$$\forall 1 \le i \le k, |S_i| \le 5. \qquad (2)$$

Combining equation (1) and (2), we get

$$|S| \le \sum_{i=1}^{k} |S_i| \le 5k. \qquad (3)$$

The size of S is at most 5 times of the minimum WCDS. ∎

Theorem 2. *The message complexity of the algorithm is $O(n)$.*

Proof. During the algorithm, each node sends out at most two messages. So the number of messages is at most $2n$. The theorem is yielded. ∎

Moreover, our algorithm requires only single-hop neighborhood knowledge and the length of the messages is $O(1)$. So it is practical to be implemented and the communication overload is small.

4. CONCLUSION

In this paper we propose a distributed algorithm to construct a weakly connected dominating set for mobile ad hoc networks, which can be used as a virtual

backbone. Our algorithm outperforms those existing algorithms in term of approximation factor and message complexity. We are currently evaluating our algorithm by experiments to test its average case performance.

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